Design Guidelines for Ductility and Drift Limits

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ABSTRACT AND ACKNOWLEDGEMENTS

This report summarizes each of the studies that have been conducted in California as a part of the CURIE-Kajima Research Project #5, entitled "Design Guidelines for Ductility and Drift Limits." This research project has been supported by a grant provided by the Kajima Corporation and administered by CURIE (California Universities for Research in Earthquake Engineering). This financial support is gratefully acknowledged.

The report consists of seven chapters. The first six chapters summarize the six different studies that have been conducted according to the agreed team research project plan. These studies are described in detail in the seven CURIE-Kajima reports given below.

REPORTS


Chapter 7, after a brief review of the studies reported in the above seven reports, (summarized in the first six chapters), presents guidelines for the development of a reliable method for estimating the values of response reduction factor R and discusses how these values could be used to improve present U.S. and Japanese code procedures for earthquake resistant design.

This report summarizes only the work done by researchers of the CUREe team. The valuable contributions of the Kajima team to this joint research project are recognized and gratefully acknowledged.
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EARTHQUAKE RESPONSE AND ANALYTICAL MODELLING OF THE
JAPANESE S-K BUILDING

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Inelastic Analysis of S-K Building
Conclusions.
DESCRIPTION OF BUILDING

The (S-K Building) is a 30 story concrete apartment building located near Tokyo and constructed by the Kajima Corporation in March 1987.

Figure 1 shows floor plans of the building. Figure 2 shows sections through the building at grid lines A and C. The strength of the concrete varies along the height of the building as shown.

The foundation of the building consists of 3m thick girders along the column lines, piles beneath the interior columns and a continuous bearing wall on the exterior of the building. The bottom 1.3m of the piles and exterior bearing wall penetrate a layer of alluvial sandy gavel at about 32m below ground level.

Instruments were placed at the roof, ground floor and 35m below ground level (beneath the deep foundation).

DESCRIPTION OF EARTHQUAKES

Two earthquakes were considered in the study - the Chiba To-Ho Oki earthquake and the Tokyo To-Bu earthquake.

The Chiba To-Ho Oki earthquake occurred on December 17, 1987 at 11:08 JST. It had a Magnitude of 6.7 and its hypocenter was at 35.2N, 140.29E at a depth of 58Km. At the ground floor of the building a peak acceleration of 0.04g was recorded during the
earthquake. Figure 3 shows the 5% damped response spectrum of the ground motion at the base of the building in the x and y directions compared to two popular California design earthquakes, EQ-I and EQ-II [1]. EQ-I corresponds to an earthquake with a fifty percent chance of being exceeded in fifty years or a chance of occurring once every seventy two years. United States buildings are designed to remain essentially elastic up to this level of earthquake. EQ-II is the maximum credible earthquake and has a ten percent chance of being exceeded in one hundred years or a return period of nine hundred and fifty years.

The Tokyo To-Bu Earthquake occurred at 5:34 JST on March 18, 1988 and had a magnitude of 6.0. At the ground floor of the S-K building peak acceleration of 0.064g was recorded during the earthquake. The 5% response spectrum of the ground motion is also compared with EQ-I and EQ-II in Figure 4.

Since ground motion at the base of the building during both earthquakes was much smaller than EQ-I, no substantial inelastic behavior of the building is expected to have occurred during the earthquakes.
Figure 1  Typical floor plans in S-K building.
Figure 2  Section through frames in S-K building.
Figure 3  Response spectra for Chiba To-Ho Earthquake, EQ-I and EQ-II.
Figure 4  Response spectra for Tokyo To-Bu Earthquake, EQ-I and EQ-II.
RESPONSE OF THE BUILDING

Chiba To-Ho Earthquake

Figure 5 shows the first 40 seconds of acceleration recorded at the ground floor and roof of the building in the x-direction. It can be seen that the strong ground motion starts about 20 seconds into the record, probably when the surface waves arrived at the site. (The difference in arrival times between the S-waves and P-waves is about 10 seconds). The frequency content of the acceleration at the roof is different from that at the ground indicating that the fundamental period of vibration of the building is different from the of dominating periods in the ground motion.

Figure 6 shows the acceleration recorded 35m below ground level in the x-direction for the same time. Comparing this with the acceleration at the ground floor of the building it is seen that there is an increase in acceleration and change in frequency content between the bedrock and the ground. This shows that the 32m deep layer of soft soil between the ground floor of the building and the sandy gravel on which the deep foundation is supported has a significant effect on the ground motion experienced by the building.

Figure 7 shows the acceleration at the roof and ground floor in the x-direction later in the record. In Figure 7(a) a fundamental period of approximately 1.75 seconds dominates the
Figure 5 Acceleration at ground floor and roof in x-direction (Chiba To-Ho Earthquake).
Figure 6  Acceleration 35 meters below ground level in x-direction (Chiba To-Ho Earthquake).
Figure 7 Acceleration at roof and ground floor in x-direction (Chiba To-Ho Earthquake).
response. A secondary period of vibration of about 0.47 seconds is also observed. Later in the record, with a smaller ground motion input, a period of vibration of about 1.8 seconds is noticed.

The response of the building in the y-direction is similar to that in the x-direction as seen in Figure 8. The higher ground motion also starts about 20 seconds into the record. In Figure 9 the mode with a period of vibration of about 0.5 seconds dominates the response and in Figure 10, with a smaller ground motion input, the first mode of about 1.7 seconds governs.
Figure 8  Acceleration at ground floor and roof in y-direction (Chiba To-Ho Earthquake).
Figure 9  Acceleration at roof in y-direction (Chiba To-Ho Earthquake).
Figure 10  Acceleration at roof and first floor in y-direction (Chiba To-Ho Earthquake).
A better understanding of the periods of vibration of the building is obtained by the use of response spectra. The response spectrum for a particular motion is a plot of the maximum response that single degree of freedom oscillators of different frequencies experience when excited by the given motion. A system responds with significantly larger amplitudes when excited by a motion close to its natural period of vibration (resonance). Thus a response spectrum for a given acceleration record will be characterized by peaks at frequencies that dominate the response of that part of the building.

Figure 11(a) and Figure 11(b) show the acceleration response spectra in the x and y-directions, respectively of the roof, ground floor and 35m below ground floor. Twenty seconds of the record – the time between 20s and 40s was used in calculating the spectra since it is during this time that the highest acceleration was experienced by the building. The response spectra were calculated using the program SRS2 developed by Professor Gerard Pardoen of the Department of Civil Engineering, University of California, Irvine on the spreadsheet LOTUS 123 [2].

The x-direction, shown in Figure 11(a), shows peaks in the roof response at frequencies of about 0.57Hz (1.75s), 1.3Hz (0.79s) and 2.3Hz (0.44s). The response spectrum of the ground floor shows a definite peak at 2.3Hz (0.44s) and this is probably the fundamental period of the 32m deep layer of soil beneath the building.
Figure 11 Acceleration response spectra for S-K Building.
In the y-direction peaks are observed in the roof response at frequencies of 0.6Hz (1.67s), 1.15Hz (0.87s), 1.75Hz (0.57s) and 2.15Hz (0.47s).

By normalizing the response spectra calculated for the roof with that calculated for the ground floor, the peaks in the roof response which occur due to ground motion of that frequency are filtered out. This spectral response ratio is shown in Figure 12 for both directions. From the figure the first mode in both directions is seen to have a period of 1.67 seconds. The second mode in the x-direction has a period of 0.5 seconds and the second mode in the y-direction has a period of 0.48 seconds.

Tokyo To-Bu Earthquake

Figure 13 shows the first 40 seconds recorded at the ground floor and roof of the S-K building during the Tokyo To-Bu Earthquake in the x-direction. The highest acceleration is observed 20 seconds into the record. The difference between the response at the ground and the response 35m below the ground, shown in Figure 14, indicates that once again soil between the ground floor and the bedrock magnifies the ground motion.

In Figure 15 the acceleration at the roof in the y-direction is shown. A period of vibration of 0.52 seconds is observed.

A plot of the ratio of the spectral acceleration at the roof and the spectral acceleration at the ground floor is shown in Figure 16. A fundamental period of vibration of 1.75 seconds is
Figure 12  Ratio of spectral acceleration at roof an spectral acceleration at ground floor.
Figure 13  Acceleration at ground floor and roof in x-direction (Tokyo To-Bu Earthquake).
Figure 14  Acceleration 35 meters below ground level in x-direction (Tokyo To-Bu Earthquake).
Figure 15 Acceleration at roof in y-direction (Tokyo To-Bu Earthquake).
Figure 16  Ratio of spectral acceleration at roof and spectral acceleration at ground floor (tokyo To-Bu Earthquake)
observed in both directions. In the x-direction the second mode is 0.5 seconds and in the y-direction the second mode is 0.53 seconds.

**COMPUTER ANALYSIS**

A three dimensional computer model of the S-K building was developed using the finite element analysis program SAP90 [3]. Time history analyses were performed the Tokyo To-Bu and Chiba To-Ho Earthquakes.

**Description of the Model**

The building was modelled using beam-column elements for the columns and beams in the moment resisting frames. Figure 17 shows the outline of the model.

Since the ground motion at the site during the earthquakes was well below the EQ-I it was assumed that the forces the building was subjected to were far less than the yield capacity of the members and that cracking of the concrete was minimal. Thus, the gross sectional properties of the beams and columns were used in the properties of the elements.

The elastic modulus of the concrete was used and this varied with the strength of the concrete with the equation [4]:

\[ E_c = 57000\sqrt{f_c} \text{ psi} \]
Figure 17  Outline of SAP90 computer model.
The floors were assumed to be rigid diaphragms and the mass at each story was concentrated at the center of mass of the floors.

The network of 3m deep beams and that connected the ground floor to the piles and bearing wall was considered to be very rigid and so the structure was modelled as being fixed at the ground level.

In the time history analysis, the first ten modes of the building were considered and a damping ratio of 5% was used for all modes. This damping ratio of 5% corresponds to the value recommended in Table 4.1 of the "Seismic Guidelines for Essential buildings" by the Joint Departments of the U.S Army, Navy and Airforce [1] for structures that resist forces with elastic or nearly elastic behavior. The peak acceleration levels at the site of 0.04g and 0.064g for the Chiba To-Ho Oki Earthquake and the Tokyo To-Bu Earthquake, respectively should result in nearly elastic response of the building. The results of systematic studies of buildings [11] also indicate that a damping ratio of 5% is acceptable.

Mode Shapes

The first ten periods of vibration of the structure calculated from the analysis are shown in Table 1.

The shapes of the first and second modes are illustrated in Figure 18. The modes involve a translation along the "diagonals" of the building but while the first mode also exhibits some
rotation, the rotation in the second mode is negligible. The presence of rotation in the first mode explains the difference in period of the two modes. (1.68s and 1.32s).

Figure 19 describes the third and fourth modes. Both shapes are the second translation modes along the diagonals of the building but the third mode exhibits some rotation. The effect of the rotation is less in this secondary mode and thus the two periods are closer. (0.67s and 0.66s).

The fifth mode is essentially a pure rotation mode.

<table>
<thead>
<tr>
<th>FREQUENCY (CYCLES/SEC)</th>
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<tbody>
<tr>
<td>1</td>
<td>0.59</td>
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<tr>
<td>2</td>
<td>0.75</td>
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<td>3</td>
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<td>4</td>
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<td>9</td>
<td>3.98</td>
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<td>5.34</td>
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Figure 18  First and second modes obtained from computer analysis.
(a) Translation of center of mass

3rd mode

4th mode

(b) Plan view of mode shape at roof.

Figure 19  Third and Fourth modes obtained from computer analysis.
The sixth and seventh modes are the third translational modes and the eighth and ninth mode are the fourth translational modes. As in the first and third mode, the sixth and eighth modes exhibit some rotation. The tenth mode is the fifth translational mode along a diagonal.

Time History Analyses and Response Spectra

Time history analyses were performed on the SAP90 computer model using the acceleration recorded at the ground floor of the building as the base excitation. This was done to validate the accuracy of the model.

Figure 20 and Figure 21 compare the measured and calculated response at the roof of the building in the x and y-directions respectively, for the 20s - 40s period of the Chiba To-Ho Earthquake. The comparison is favorable. The time history curve for measured response is smoother and this is expected since the elastic computer analysis cannot take into account the slightly non linear response that results from cracking, rubbing of nonstructural elements members and small amounts of yielding.

In Figure 22 the spectral response ratios obtained from the computer analysis are compared with those from the measured records. The first mode compares favorably but the second translational mode differs by about 0.15 seconds.

A similar result for the Tokyo To-Bu earthquake is shown in Figure 23.
Figure 20  Comparison of recorded and calculated response of building in x-direction.
Figure 21  Comparison of recorded and calculated response of building in y-direction.
Figure 22  Comparison of ratio of spectral acceleration at roof and spectral acceleration at ground floor. (Chiba To-Ho Earthquake).
Figure 23 Comparison of ratio of spectral acceleration at roof and spectral acceleration at ground floor.
(Tokyo To-Bu Earthquake).
INELASTIC ANALYSIS OF S-K BUILDING

In this section an inelastic analysis is performed on the S-K building to determine the post yield capacity or ductility of the building. This is important since large earthquakes subject buildings to displacement beyond the elastic limit.

A two dimensional model of a typical moment resisting frame in the building is used. This two dimensional model was developed using the properties of the three dimensional model used in the previous section.

The inelastic analysis of the structure was approximated by using an elastic step-by-step superposition analysis. This involved applying a steadily increasing static triangular load (as shown in Figure 24, with a point load applied at each floor) to the structure. When the moment on a member reaches its plastic moment capacity, the structure is modified by inserting a hinge at the point where the plastic moment was reached. The load is then increased until another hinge forms. This process is continued until enough hinges are formed to cause instability in the structure.

A shell control program that performs this method of inelastic analysis in conjunction with SAP90 has been developed by Shimano [6] and is used in the study.
Figure 24  Triangular lateral load applied to S-K Building.
Capacity of Members

In the hinge zones of the column and beams of the S-K building, closely spaced lateral reinforcement was used to provide confinement of the concrete. This confinement affects the behavior of the concrete significantly. Thus, in calculating the capacity of a member (moment at which a hinge will form), a stress-strain curve that adequately describes the behavior of the confined concrete needs to be used. The stress-strain curve developed by Mander, Priestley and Park [7] was used in this analysis. This stress-strain curve is determined by the effective confining pressure provided by the lateral reinforcement on the concrete.

The stress-strain curve of the concrete and thus the capacity of a member depends on four variables:

(a) The compressive strength of the concrete.
(b) The dimensions of the cross section of the member.
(c) The amount, arrangement and strength of the longitudinal reinforcement.
(d) The amount and strength of lateral reinforcement.

The effect of the compressive strength and dimensions of the cross section on the capacity of the members are obvious - higher compressive strengths and larger sections result in greater moment capacity.

The longitudinal reinforcement affects the capacity of the member not only by determining the ultimate tensile force and moment arm, but also by influencing the stress-strain curve used.
for the concrete. The area of effectively confined concrete is reduced by arching action between the longitudinal bars. This decrease in effectively confined concrete area by arching reduces the confined compressive strength of the concrete and thus affects the stress-strain curve used in the analysis. When the longitudinal bars are placed further apart the arching takes place over a longer distance. The area of effectively confined concrete is decreased and so the capacity of the member is lower.

The lateral reinforcement affects the strength of a member in a similar manner since arching action also occurs between the ties or spiral reinforcement. Therefore, a larger spacing or pitch of the lateral reinforcement results in weaker members. Also, the size and strength of the lateral reinforcement determines the confining pressure provided.

These four variables change significantly in the S-K building and so stress-strain curves were calculated for the members with different properties. The moment capacities of the members were then calculated using the computer program IMFLEX (Hart, Sajjad and Basarkhah [8]). The program is capable of calculating moment-curvature diagrams for concrete and masonry beams with any arrangement of longitudinal steel and a specified stress-strain curve for the concrete or masonry. A more detailed explanation of the calculation of the stress-strain curve and moment capacity of members in the S-K building has been provided in a previous paper [9].

A typical moment-curvature diagram for a beam is shown in
Figure 25. Yielding of the longitudinal steel is seen to govern the plastic moment capacity. The columns and beams of the S-K building possess very high ductility and concrete strain did not control the plastic moment in any of the members. This is particularly so since confined concrete can endure strains of about 0.03 [7]. The unconfined cover concrete was assumed to spall at a strain of 0.005 as recommended by Scott, Park and Priestley [10].

No strain hardening is accounted for in the analysis. The members were assumed to be perfectly plastic. The floor slab was assumed to be infinitely rigid and so the beams did not carry any axial load. The moment-curvature diagrams for the columns were calculated for the axial load that they were subjected to from dead loads and lateral loads.

Results of Analysis

Figure 26 shows the load-deflection curve calculated from the analysis. The graph is normalized by plotting the total drift ratio against the base shear coefficient. The total drift ratio is the displacement at the roof divided by the total height of the building. This gives an idea of the average story drift in the building.

The first hinge occurred at a base shear coefficient of 0.11 and a total drift ratio of 0.23 percent. This indicates a relatively stiff building since the allowable inter-story drift ratio at yield is about 0.5 percent for U.S. buildings. The ultimate total drift ratio when the structure becomes unstable
Figure 25  Typical Moment-curvature diagram for member
Figure 26  Load-deflection curve for typical S-K building frame. (Gross section properties).
was 0.46 percent. Thus, the displacement ductility of the building is

\[ \mu = \frac{\Delta_u}{\Delta_y} = \frac{0.46}{0.23} = 2.0 \]

This means that the building can endure displacements twice those for which first yield occurs. It should be noted that this is the ductility of the building as a whole. Components of the building may possess greater or less ductility.

To account for the reduction in the moment of inertia in the members caused by cracking of the concrete, the analysis was repeated using eighty percent of the gross moment of inertia in the columns and 40 percent of the gross moment of inertia in the beams - a common approximation. The load-displacement curve is shown in Figure 27. The total drift ratio at yield was 0.44 percent and the total drift ratio at instability was 0.87 percent. This gives a ductility of 1.97 or approximately 2.0 - the same value calculated previously. Also, the shape of the load deflection curve in Figure 27 is similar to that in Figure 26. This suggests that even though the deflection depends on the moments of inertia selected, the calculated ductility does not.

In reality, it is expected that during the loading process the moments of inertia would decrease gradually as more cracking takes place in the members with the increasing load. Hence, the deflections at and instability would be somewhere between the values calculated for the two cases. Thus 2.0 serves as a lower
Figure 27  Load-deflection curve for typical S-K building frame.  
\( (E_{\text{col}} = 0.8E_{\text{gross}}; \ E_{\text{beam}} = 0.4E_{\text{gross}}). \)
bound for the ductility. An upper bound can be obtained by taking
the ratio of the ultimate deflection for the second case and the
deflection at first yield for the first case (gross moments of
inertia). This would give a value of

$$\mu_{upper} = \frac{\Delta u(\text{reduced})}{\Delta y(\text{gross})} = \frac{0.87}{0.23} = 3.78$$

The S-K building can then be said to have a ductility of between
2.0 and 3.8.

Figure 28(a) and Figure 28(b) show the sequence of formation
of the 246 hinges that were calculated in sets of 40. The
building is seen to obey the "strong column weak beam" concept
with all the hinges forming in the beams. This ensures that
premature instability does not develop. Plastic hinges are seen
to form first around the eighteenth floor and then around the
seventh floor. This is because the nominal strength of the
concrete is reduced at these floors causing a sudden drop in the
moment capacities of the beams on these floors relative to the
floor below.
Figure 28(a) Sequence of hinge formation in inelastic analysis.
Figure 28(b) Sequence of hinge formation in inelastic analysis.
CONCLUSIONS

1. The first and second translational periods of the building in are approximately 1.7s and 0.5s.

2. The computer model describes the building reasonably well with translational modes of 1.68s and 0.66s respectively. Further adjustment of the parameters of the building by a more rigorous calculation of the building properties would make the comparison to the real structure even more accurate.

3. The outline of the building in plan is approximately square and thus the principal axis are along the diagonals of the building. The translational modes of vibration thus involve motion along these diagonals. This explains the similarity in recorded response in the x and y-directions. The position of elevator shafts and the shape of the floor plans in the building leads to a slightly non-symmetrical arrangement of mass on the floors. This eccentricity is the reason for the coupling of rotation and translation in some modes of vibration of the building. Figure 29 shows the approximate positions of the center of mass for some floors. In all cases the eccentricity from the x-axis is greater than that for the y-axis. The eccentricity from one diagonal is also greater. Thus the results from the analyses showing significant coupling of translation and rotation in
one direction and negligible coupling in the other are expected.

The measured response also show similar behavior. Figure 7 shows a first mode period of about 1.8 seconds in the x-direction and figure 10 shows a period of about 1.67 in the y-direction. Figure 12 and Figure 16 also show that there is a slight difference in period for the first two modes in the y-direction and the first two modes in the x-direction.

4. The building exhibits reasonably strong ductile behavior. It adheres to the "strong column weak beam" concept with plastic hinges forming in the beams and not in the columns. The building can be said to possess an overall ductility of between 2 and 4.
SECOND FLOOR PLAN

Figure 29(a) Position of center of mass on floor plan.
Figure 29(b) Position of center of mass on floor plan.
REFERENCES


Displacement Design Approach
for Reinforced Concrete Structures
Subjected to Earthquakes

by
Xiaoxuan Qi
and
Jack P. Moehle
ABSTRACT

An analytical study of the characteristics of inelastic displacement response of single-degree-of-freedom (SDOF) systems subjected to earthquake ground motions was conducted to develop practical means of estimating the peak values of the displacement. It is observed that the displacement response can be characterized in two period ranges, divided by the characteristic period of the ground motion. Methods are established by which elastic analysis can be used to estimate the peak displacement response in both period ranges. Therefore, a design displacement response spectrum can be constructed for a given earthquake. The spectrum provides a measure of the expected inelastic displacement using a linear model.

The method of estimating the displacement of inelastic SDOF systems is extended to estimate the peak lateral displacement of multi-story frame structures. The maximum inter-story drift that a frame could experience during a given earthquake is approximated based on the estimated roof level displacement.

A study of the deformation capacities of reinforced concrete frame members and sub-assemblages is conducted to relate the details provided in reinforced concrete beams and columns to the maximum inter-story drift capacity. It is concluded that the required details for reinforced concrete members can be assessed based primarily on displacement considerations.

A new displacement-based seismic design approach is outlined. Design examples demonstrate that the displacement response of structures can be effectively controlled using this approach. It is concluded that more efficient control of the seismic performance of buildings can be achieved using the displacement design approach. The importance of structural stiffness in resisting earthquake excitations is highlighted in the application of the displacement approach.
This report is based on work conducted by the first author as part of his studies toward the doctoral degree at the University of California at Berkeley under the supervision of the second author.

Facilities of the Department of Civil engineering and the Earthquake Engineering Research Center at the University of California were used to conduct the studies.

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CHAPTER 1

INTRODUCTION

The response of buildings to earthquake ground motions has been one of the major concerns in modern structural design and construction in regions of high seismicity. An extensive effort has been spent in the past thirty years to understand the characteristics of earthquake ground motions, and the characteristics of structural responses to ground motions. Many design methods [eg. 2,53,63] have been developed to guide the earthquake-resistant design of buildings and other structures. As knowledge in the area of earthquake engineering develops, it is expected that modern structures can be designed and constructed to withstand strong earthquake ground motions with greater reliability. Nonetheless, there remain many uncertainties in modern earthquake engineering design. One basic area in which great uncertainty remains is the evaluation and control of structural damage. Control of structural displacement is an essential element of damage control. The goal of the current study is to develop techniques to estimate structural displacements during strong earthquakes, techniques to relate expected displacements to required structural details, and a design algorithm that incorporates these techniques.

1.1 Expected Seismic Response for Engineered Buildings

As part of the current study a survey was conducted of a group of structural engineers in California (in the San Francisco Bay Area and the Los Angeles Area). The survey was designed to collect information about expected reinforced concrete building performance during earthquakes. The engineers that were surveyed are believed to be well experienced in earthquake-resistant design of building structures, and to be knowledgeable in the current state of the art in earthquake-resistant design.

The questions posed in the survey were general, and were expected to be answered based on the personal sense and knowledge of the engineer rather than current accepted
and published criteria. The details of the questionnaire used in the survey can be found in App. A. Approximately seventy percent of the surveyed engineers responded. A summary of the responses is presented in Fig. 1.1. Considering answers with regard to earthquakes having 50-year return period, approximately 60% of the engineers expected that buildings would suffer only light damage; 20% of them did not expect any damage at all; and the other 20% would accept some serious damage. Because the majority of those engineers who responded expected no more than light damage, a general question rises naturally: How should buildings be designed and proportioned in order to prevent them from suffering serious damage?

At the present time, the state of the practice for earthquake-resistant design can be considered as outlined in the Uniform Building Code (UBC) [63]. According to the UBC, the effects of an earthquake on regularly configured structures are considered through a set of equivalent static lateral loads acting at floor levels. Structural members are proportioned in accordance with the strength demand based on elastic analysis under various loading conditions, including the equivalent earthquake static loads. Elastic displacements are usually examined under the design earthquake lateral loads, and are not to exceed a code specified limit. There is no direct assessment of the magnitude of inelastic displacement that the structure may experience during the design earthquake, or of the structural or nonstructural damage that might result.

In a study conducted by X. Qi, and subsequently reported in reference [29], the relation between the elastic lateral displacement under UBC [63] specified design lateral loads (designated as design drift) and the numerically computed [33] dynamic displacement response under recorded earthquake ground motions (designated as computed drift) was examined. The study was performed on typical reinforced concrete structural systems and for earthquakes having different characteristics. It was demonstrated that the ratio of computed drift response and design drift varied from case to case, and there was no simple bound that could be easily identified (Fig. 1.2). However, the computed drift always exceeded the design drift, and in most cases the ratio of calculated to design drift exceeded five.
The deformation of a structure has been recognized as the centerpiece in evaluating the performance and damage state of structures [1,19,31,32]. If the damage state of a building during an earthquake is considered to be directly related to the deformation that building experiences, then clearly it is important to be able to estimate the drift during the design stage. As indicated in Fig. 1.2, currently accepted design methods do not provide a good estimate of actual drift. Design methods that will provide a drift estimate, and consequently indicate the damage state, are highly desirable.

1.2 Objectives and Scope

The objectives of the current study are to establish relations between the drift response of buildings and the behavior of structural members, to develop techniques by which the displacement response (elastic or inelastic) of regularly shaped building structures under earthquake ground motions can be estimated, and to incorporate those techniques into a displacement–based earthquake–resistant design procedure. The study is oriented toward applications to reinforced concrete moment resisting space frames, but is applicable to other structural types and materials.

The deformation capacities of members of reinforced concrete frames are first studied in Ch. 2 and Ch. 3. Methods that can be used to estimate the dynamic displacement responses of idealized single–degree–of–freedom and multi–degree–of–freedom systems are developed in Ch. 4 and Ch. 5, respectively. Finally, a displacement–based earthquake–resistant design approach is outlined in Ch. 6, and examples using this approach are presented.
CHAPTER 2

REVIEW OF EXPERIMENTAL STUDIES
ON REINFORCED CONCRETE STRUCTURES

Experimental study has been one of the major resources in understanding the behavior of reinforced concrete members and structures. Many tests have been conducted in the last few decades at various universities and research institutes, and different aspects of the behavior of reinforced concrete components and structural systems have been addressed. Among those tests, some were designed to study the general behavior of structural members or of complete structural systems, whereas others focused on very specific issues such as effects of transverse steel on performance. In the present study it is of interest to examine results of experiments in terms of deformation capacity. Although the apparent effects of some of the main variables will be discussed, no attempt will be made to analyze these effects in detail. Rather, the object is to establish an approximate lower bound estimate of available deformation capacities as obtained in laboratory tests of reinforced concrete elements. The results will subsequently be used to set a target upper bound design drift for reinforced concrete structures subjected to earthquakes.

Many tests reported in the literature were conducted on small scale specimens. Small scale model tests provide a powerful and economic tool to understand the behavior of structures. However, there are uncertainties introduced by using small scale specimens. Particularly, small scale models may not be constructed in the same way as their prototype structures built in the field and details of their behavior may not be totally representative of the behavior of the prototype structures. Therefore, large scale specimens are generally preferred for the purpose of assessing details of the behavior of real structures. The present study will be limited to tests of large scale models (at least 50 percent of full scale) having material properties that fall into the range of common practice. The reviewed experiments are divided into three groups, namely beams, columns, and beam-column assemblages.
2.1 Tests of Reinforced Concrete Beams

Beams are defined as structural members subjected primarily to flexure and shear. The effects of axial load acting on beams are small enough to be neglected. Two typical setups for conducting beam tests are shown in Fig. 2.1. In the cantilever beam test setup (Fig. 2.1a), one end of the test beam is “fixed” to a concrete block or to a short column stub (the deformation of the column stub is negligible). The other end is free to move in the direction of applied load and can undergo rotation about an axis perpendicular to the plane of the beam. This test setup simulates the portion of a beam in a two-dimensional frame from the face of a column to the inflection point in the beam, which is usually modeled as occurring at the beam mid-span. In the symmetric beam test setup (Fig. 2.1b), there are effectively two beams in one specimen. By displacing the column stub at the center in a transverse direction, loading conditions similar to that in cantilever beam tests are developed on both sides of the column stub. The column stub is built to simulate a beam-column connection and to apply the lateral load. Therefore, each side of the specimen is equivalent to a cantilever beam except the slippage of longitudinal reinforcement within the joint tends to be reduced in the symmetric beam test due to symmetric deformation.

A concentrated force is applied at the free end of the cantilever beam (or at the center column stub of a symmetric beam test) to impose lateral displacement reversals in the plane of the beam. The applied force results in a constant shear force along the test beam (ignoring beam self weight) and a bending moment that increases linearly towards the fixed end. Several different types of load histories have been investigated in the reported tests. In the present study only beams subjected to cyclic loads are considered. For some tests the cyclic loading was at a constant amplitude that was of sufficient magnitude to fail the specimen within a few cycles. In others, the amplitude was increased gradually as the test progressed. In the latter test type, small amplitude cycles may have been induced between large amplitude cycles.

The deformation capacity of a test beam is normalized as the measured peak deflection at the loading point divided by the length of the beam measured from the face of the column.
stub, and is designated as the equivalent beam end rotation, $\theta_e$. Because degradation of resistance of a specimen is generally associated with large deformation reversals, and because excess degradation may be associated with overall loss of structural stability in some cases, it is common practice to define deformation capacity as the deformation attained prior to loss of some percentage of the maximum resistance. In the current study the beam deformation capacity is defined by the maximum transverse displacement that is sustained by the beam without losing more than 15% of the maximum resistance. Several tests were apparently terminated prior to the occurrence of significant deterioration in resistance. For these tests, the equivalent beam end rotation reported here is the rotation corresponding to termination of the test, and may not be a true measure of the actual capacity.

In a building having rigid columns, the equivalent beam end rotation at any stage of loading would be approximately equal to the inter-story drift index (defined as the lateral inter-story displacement divided by the story height). Because in a real building frame the column also contributes to the total deformation, the equivalent beam end rotation capacity is effectively a lower bound measure of the maximum inter-story drift index that could occur in a frame assuming premature column or joint fracture does not preclude attainment of beam deformation capacity.

Numerous variables have been considered in previous experimental studies of reinforced concrete beams [6,8,15,23,44]. The following quantities are selected to characterize the response of the beams cited in this review:

1. Reinforcing index, $r_i = \rho f_y / f'_c$, where $\rho$ is the main longitudinal steel ratio ($A_s/bd$), and the stresses $f_y$ and $f'_c$ are the yielding strength of the reinforcing steel and the cylinder compressive strength of the concrete, respectively. In defining $\rho$, $b$ is the beam width, $d$ is the beam effective depth, and $A_s$ is the maximum of either the top or bottom longitudinal steel areas.

2. Ratio of minimum to maximum longitudinal reinforcement, $\rho' / \rho$. The quantity $\rho'$ is equal to $A'_s/bd$, in which $A'_s$ is the minimum of the top and bottom reinforcement ratio. The ratio $\rho'/\rho$ varies from 0.5 to 1.0 in the beams reviewed.
(3) Transverse steel index, $\rho_s \sqrt{b/s}$, where $\rho_s$ is the ratio of the volume of transverse steel to the volume of concrete core enclosed by the transverse steel [43], and $b$ and $s$ are the width of the beam web and the center to center spacing of transverse reinforcement, respectively.

(4) Shear span ratio, $L/d$, where $L$ is the length of the test beam measured from the face of column stub to the beam inflection point. The ratio provides an indication of the relative intensity of shear force and bending moment acting on the critical regions.

(5) The history of applied loads or induced displacement at the loading point. As noted previously, two types of loading history are considered; cyclic loading at constant amplitude, and cyclic loading at increasing amplitude.

Thirty-six beams tested at four different universities are cited in the present study. Typical measured load–deformation relations are plotted in Fig. 2.5. The values of the characteristic quantities described above for all thirty-six beams and their normalized deformation capacities, defined in terms of the equivalent beam end rotation capacity, are summarized in Table 2.1. It is observed that peak equivalent beam end rotation capacity of these beams varies from 0.0276 to 0.1424 rad. Factors that may affect the deformation capacity of reinforced concrete beams are discussed and evaluated using the test data in the following paragraphs.

The load history has a pronounced effect on the observed deformation capacity. Under cyclic loadings, progressive damage to the beam due to reversed displacement reduces the deformation capacity as compared with beams tested under monotonically increased loading [8]. Furthermore, beams subjected to constant large magnitude displacement cycles were generally able to undergo larger peak displacement than beams subjected to cyclic loading histories with progressively increased displacements (Fig. 2.6). However, when subjected to constant large magnitude cycles, fewer cycles were usually observed prior to failure. Because load history has significant effect on behavior, and because each experimental program had different prescribed loading histories for different specimens, it is difficult to discern the effects of other variables on response.
A variation of deformation capacities associated with different transverse steel indices and shear span ratios was noticed from the experiments (Fig. 2.6b, d). In order to isolate the effects on deformation capacity of these two parameters, normalized beam end rotation capacities corresponding to those beams having equal amount of top and bottom reinforcements and subjected to lateral displacement cycles with gradually increasing magnitude are shown in a three dimensional plot (Fig. 2.7) versus transverse steel index and shear span ratio. Despite a paucity of data in some regions of the plot, it is observed that the deformation capacity tends to increase as shear span ratios become larger and as transverse reinforcement is increased.

In the tests reviewed, the quantity $\rho'/\rho$ usually had one of two values, either approximately 0.5 or 1.0. The effects of variation in this ratio are not apparent (Fig. 2.6c).

The value of the longitudinal reinforcing index varied from 0.0913 to 0.2742. If normalized with respect to Grade 60 steel and 4000 psi concrete, these values correspond to steel ratios $\rho$ between 0.61% and 1.83%. No pronounced effects of the variable $\rho$ on deformation capacity are observed from the test data (Fig. 2.6a) though the lowest deformation capacities were obtained for the highest longitudinal reinforcing indices.

### 2.2 Tests of Reinforced Concrete Columns

Columns are defined as structural elements subjected to combinations of bending moment, shear, and axial forces. In order to simulate the three kinds of loadings in tests of isolated column units, different test setups have been adopted in various experimental studies (Fig. 2.2). In the fixed-fixed column test setup (Fig. 2.2a), a column having both ends fixed is tested, instead of a portion from the fixed end to the point of inflection. Cantilever and symmetric test setups for columns are similar to their beam counterparts (Fig. 2.2b, c), except axial loads are applied to the column unit.

In most tests, the axial compression load is first applied to a column and kept constant throughout the test while lateral displacement is imposed. A constant compression force simulates the loading conditions for a typical interior column in a frame structure,
where the axial load is considered not to change much due to lateral loads. In some tests, columns are subjected to variable axial loads along with lateral load, simulating the situation of an exterior column where the axial load varies due to overturning moment. All columns reviewed herein were square columns with symmetric longitudinal reinforcements. The imposed lateral displacement reversals were either fixed in one direction (parallel to a principal axis or to a diagonal of the column section) with gradually increased magnitudes; or varied in both principal directions following pre-defined displacement paths. Some tests apparently were terminated before reaching maximum deformation capacity.

The same technique used in normalizing the deformation capacity of beams was used in evaluating the behavior of columns. The equivalent column end rotation was determined as the peak measured lateral displacement divided by the length of the column segment. It has a similar relation to the inter-story drift index as does the equivalent beam end rotation for beams. In those cases for which the imposed lateral displacements were not parallel to a principal axis, the larger component along a principal axis was used in the calculation of equivalent column end rotation.

In addition to different axial loading histories and different lateral displacement paths, other variables used in the experimental studies were as follows:

1. Longitudinal reinforcing index, \( r_i = \rho_t f_y / f'_c \), where \( \rho_t \) is the total area of reinforcing steel divided by the gross section area of the column, \( A_g \).

2. Axial load ratio, \( P / A_g f'_c \), where \( P \) is the maximum axial load applied to the column. A positive value of \( P \) represents a compression force.

3. Transverse steel index, the same as defined for beams.

4. Shear span ratio, the same as defined for beams.

Typical measured load deformation relations, expressed in terms of normalized lateral load and column end rotation, are presented in Fig. 2.8. The values of each primary variable and the equivalent column end rotation capacities for the columns reviewed [17,18,46,48,70]
are listed in Table 2.2. The variations of the column end rotations with respect to each primary variable are shown in Fig. 2.9.

Practically, none of the variables individually showed a predominant effect on the deformation capacity of the columns, except columns having a small shear span ratio \((L/d = 1.73)\). Those columns suffered shear damage [46], which was different from that occurring in the others, and showed consistently less deformation capacity than those having larger shear span ratios. Focusing on the columns having shear span ratios greater than 2.0 (where flexural behavior is likely to be dominant) and subjected to lateral displacement along one principal axis of the cross-section, the effects of axial load ratio and transverse steel index may be better observed in Fig. 2.10. It is noticed that under uniaxial lateral displacements if the shear span ratio is larger than 2.0, and if the axial load ratio is less than 0.4, columns are likely to be capable of undergoing equivalent column end rotation of 0.02 rad with only limited loss (less than 15%) of their load carrying capacities. In cases that lateral displacements were imposed along the diagonal or along both principal axes, the deformation capacity (in the direction parallel to a principal axis) was slightly reduced.

2.3 Tests of Reinforced Concrete Beam-Column Assemblages

Typical experimental setups for tests of beam-column assemblages are shown in Fig. 2.3 and 2.4. Axial loads may be applied to the column and lateral displacement reversals are imposed at either the end of the beam (not shown) or the end of the column (as shown in the figures).

The deformation capacity of beam-column assemblages can be expressed directly in terms of the drift index, which is defined as the peak displacement divided either by the length of the column (in cases that lateral displacement is imposed at the end of the column, as shown in Fig. 2.3 and 2.4) or by the length of the beam (in cases that lateral displacement is imposed at the end of the beam). The deformation capacity is defined as the maximum value reached during the test without losing more than 15% of the maximum lateral load carrying capacity of the specimen. The drift index so defined can be considered as being
the average of inter-story drift indices above and below the beam-column joint.

There are many factors that affect the behavior of beam-column assemblages. Any quantity that affects the behavior of beams and columns will affect the performance of an assemblage composed of these elements. Furthermore, some additional variables are introduced in a beam-column assemblage, such as the relative strength and stiffness of beams and columns; the details provided inside the joint region (the behavior of joint regions was not considered in tests of beams and columns); and the existence of transverse beams and floor slabs and the variations of their sizes.

Figure 2.11 shows typical responses of interior and exterior beam-column assemblages having details typical of those used in modern construction in regions of high seismic risk. Table 2.3 lists thirty-six specimens tested at different institutions [5,9,13,14,21,22,71]. For each test, only the overall configuration of the specimen and the lateral load condition are presented together with the drift index. Details used in each assemblage varied, but for the most part, they were consistent with requirements for ductile frames. It is difficult to draw conclusions as to the effects of different details and variables. A general study of those factors is not pursued here. Instead, only drift indices that were calculated from measured deformation capacities are of interest. It is noticed that the value of drift indices varies between 0.0372 and 0.1257, and thirty-four out of thirty-six specimens sustained successfully a drift index of 0.04 or more. Connections developing drift capacities below the value 0.0372 were not identified in the literature search.

2.4 Summary

It was observed in the course of reviewing the experiments of reinforced beams and columns that they were capable of undergoing equivalent beam or column end rotations of 0.02 rad or more (Fig. 2.7 and 2.9), except for a few cases of columns having very high axial compression forces or having premature shear failure. Usually, the flexural stiffnesses of beams and columns are of similar magnitude in moment resisting frame construction. Although a strong-column-weak-girder design is widely accepted in practice such that
only limited inelastic behavior should occur in columns, the distributed elastic deformation of columns and resulting beam-column joint rotations will contribute to the story drift in addition to that resulting from elastic and inelastic deformations of beams. Therefore, deformation capacity for beam-column frames can be expected to exceed that observed for isolated beams; many tests of reinforced concrete beam-column assemblages demonstrate this expectation.

The experimental data suggest that there may exist minimum deformation capacities for typical reinforced concrete structural frames, and that this capacity may provide a guide to the proportioning of frame structures. For example, all data collected in this review suggest that a frame having reasonably detailed and constructed members will be safe from structural failure if the inter-story drift ratios are kept below 0.02. The next chapter extends the findings of the experimental data on deformation capacity using analytical means.
CHAPTER 3

Parametric Study of Plastic Hinge Rotation Capacity
of Reinforced Concrete Beams and Columns

It was observed in the previous chapter that reinforced concrete beams, columns, and beam-column connections with modest axial loads and shear forces are normally able to undergo deformations corresponding to inter-story drift indices of 2\% or more in laboratory tests. With the limited set of data considered in the experimental review, it was not possible to identify clearly the effects of some of the factors on the deformation capacity. In order to clarify some of the effects, a parametric study of deformation capacities of reinforced concrete beams and columns was conducted analytically.

In the analytical study, attention was focused on the inelastic flexural deformations that occur near the ends of beams and columns. The focus was made for two reasons. First, in well-detailed and proportioned members the inelastic flexural deformations tend to constitute a major portion of the deformation components at large deformations. By considering only the concentrated flexural deformations at the member ends and ignoring other components such as distributed flexure, shear, and reinforcement slip, a lower bound estimate of deformation capacity would be achieved. Second, the flexural deformations have been relatively well studied previously, and can be expressed in relatively simple form, thus simplifying the study and the expression of its results.

The following sections describe the analytical model, compare results of the model with experimental results, and present the analytical study in which a series of beams and columns are analyzed to determine the effects of variables on their idealized deformation response.
3.1 The Plastic Hinge Model

A variety of numerical models for the plastic hinge region of a reinforced concrete member could be investigated. Despite the complexity of some current formulations, none of the models has been demonstrated to be fully capable of modeling response under reversed cyclic loadings. A simple model based on monotonic flexural response of member cross sections is used in this study. Its reliability in representing response under complex loadings will be examined by comparison with experimental results.

Figure 3.1 depicts the moment and idealized curvature distribution of a simple cantilever beam. The concentrated curvature at the member end is the deformation component of interest in the present study. By convention, the region is referred to as the plastic hinge, and inelastic curvature is idealized as spreading over the plastic hinge length, $l_p$. The inelastic rotation of this region is referred to as the plastic hinge rotation, $\theta_p$. The plastic hinge rotation can be expressed analytically in the form

$$\theta_p = (\phi_u - \phi_y)l_p$$

(a)

where $\phi_u$ and $\phi_y$ represent the ultimate and yielding curvatures, respectively.

The curvatures $\phi_y$ and $\phi_u$ are calculated using conventional procedures that assume axial strain varies linearly across the section. In the calculation, steel is assumed to have a stress-strain relation of the type shown in Fig. 3.2a. Concrete is considered to be one of two types, either unconfined concrete as may occur in the cover outside the transverse reinforcement or confined concrete as may occur within the perimeter of the transverse reinforcement. The stress-strain relations defined by Park, et al. [42] are assumed in the calculation.

The value of $\phi_y$ for a given section is defined as the curvature when steel first reaches the yield strain. The ultimate curvature $\phi_u$ is defined as the curvature when either (i) the strain of the extreme fiber of concrete in compression reaches the maximum value, $\epsilon_{c,max}$; (ii) reinforcing steel fractures in tension; or (iii) the moment carrying capacity reduces to
less than 85% of the maximum capacity. Whichever of these three conditions occurs first defines the value of $\phi_u$.

A brief survey of experimental studies of the behavior of confined concrete sections was conducted to assess the available maximum compressive strain in concrete. The tests considered in the survey [30,52,57,64] were at least half scale beam or column segments that were subjected to monotonically increasing loads. The compressive strain in the concrete was the average strain measured along a gage length on the order of the effective depth ($d$) of the member. Based on the collected data (Fig. 3.3), the linear relation between the maximum concrete strain and the confinement stress ($\rho_s f_y$) [52],

$$\epsilon_{c,\text{max}} = 0.004 + 0.02 \times \rho_s f_y$$  \hspace{1cm} (b)

was employed to determine the maximum usable compressive strain in concrete for all flexural calculations. In eq. (b), $\rho_s$ is the ratio of the volume of transverse steel to the volume of enclosed concrete core, and $f_y$ is the yield stress of transverse steel (ksi).

The plastic hinge length, $l_p$, has been considered primarily as a function of the dimensions of the member. A variety of different expressions for the plastic hinge length have been proposed [28,43,45]. Among the available expressions, the one proposed by Mattock [28] is used in this study. The length is expressed as

$$l_p = 0.5d + 0.05Z$$  \hspace{1cm} (c)

The quantity $Z$ in Eq. (c) represents the distance from the section where the maximum moment occurs to the point of inflection.

Moment–curvature relations for use in computing available plastic hinge rotation capacities were calculated using the computer program BIAX [67]. The program calculates monotonic moment–curvature responses of reinforced concrete cross sections assuming plane sections remain plane and assuming simple material stress–strain relations as shown in Fig. 3.2. The program operates by subdividing a cross section into numerous elements representing sections of unconfined concrete, confined concrete, and steel. A strain profile is
imposed across the section and the mean stress and resulting internal force on each element is solved. Iterations on the strain profile are conducted to obtain the desired solutions.

3.2 Comparison of Calculated Plastic Hinge Rotations and Measured Equivalent End Rotations

The plastic hinge model described in Section 3.1 was used to calculate plastic hinge rotation capacities of several of the beam and column test specimens described in Ch. 2. Measured dimensions and material properties were used for each specimen to the extent practicable. The calculations are restricted to those specimens having shear span ratio exceeding 2, loaded to apparent failure along a principal axis of the cross section, and subjected to load histories with gradually increasing displacement amplitudes (though several cycles at a given amplitude may have occurred). Furthermore, only beams having equal top and bottom reinforcement are included. Column tests in which two columns cantilevered from a central stub are included only if the individual column deformations were available.

Calculated plastic hinge rotation capacities for beams and columns are listed in Tables 3.1 and 3.2, respectively. Measured equivalent beam or column end rotation capacities, and ratios of calculated to measured rotation capacities are presented in the same tables.

An apparent feature of the data in Tables 3.1 and 3.2 is the relatively wide scatter in ratios between calculated and measured rotation capacities. The scatter is not unexpected given the simplicity of the numerical model in comparison with the diversity and complexity of the experimental program. The numerical model does not consider effects of elastic flexural deformation, shear deformation, and slip of reinforcement from connections. These effects may influence different test specimens to different degrees. Furthermore, loading history is not considered in the calculation algorithm. Although the specimens considered in the study had similar loading histories, there were unavoidable differences from one test to another. Some tests had one displacement cycle at each amplitude, whereas others had as many as five. Some tests increased the amplitude of groups of cycles gradually, whereas others used amplitudes that were as much as double the preceding amplitude. In the latter
type of testing program, it is inevitably difficult to define the actual deformation capacity.

Because the numerical model considers only one of several sources of deformation, if all other sources of error are excluded from consideration the ratios of calculated to measured rotation capacity in Tables 3.1 and 3.2 should all be less than unity. This expectation is realized for the column tests (Table 3.2) but not the beam tests (Table 3.1). The cause of this disparity between column and beam ratios is not known; it may arise because of differences in load history, axial load, reinforcement configuration, or other parameters. It may be noteworthy that the calculated beam rotation capacities are relatively high in many cases, and in particular for many of those cases having high ratios of calculated to measured rotation capacity. It may be unreasonable to expect a reinforced concrete beam to perform well to rotations on the order of the higher calculated values listed in Table 3.1.

Whatever the causes, the fact that several ratios in Table 3.1 exceed unity even though only plastic hinge rotations are included in the numerical calculation is not unacceptable. As noted previously, the numerical model is not a perfect replica of the reinforced concrete plastic hinge, and should not be expected to produce perfect results.

Though imperfect, the comparison between measured and calculated rotation capacities (Tables 3.1 and 3.2) indicates that there exists a correlation between the numerical model and experimental result for the range of specimens considered here. The overall trend of the correlation suggests that if the calculated rotation capacity is equal to the quantity “a”, then the actual rotation capacity will on average be nearly equal to “a”, but in extreme cases may be as low as “a/2”.

3.3 Numerical Parameter Study

The preceding section suggests that a numerical parameter study using the simple plastic-hinge model will produce imperfect but useful results. Results of a parameter study of the effects of various designer-controllable parameters on deformation capacities of beams and columns are described below.
3.3.1 Assumptions and Range of the Parameter Study

Loading History—It has been observed in the course of reviewing experimental studies of the behavior of reinforced concrete members that the load history significantly affects the load-deformation response [8]. Unfortunately, there are infinitely many possible loading histories for a real structure, and in design these cannot be well predicted or controlled. Furthermore, techniques for calculating cyclic load response of reinforced concrete elements are not developed to the point that reliable results can be ensured. Given these limitations, no attempt will be made to directly incorporate effects of cyclic loading in the analytical study. Instead, all analytical results are based on effects of monotonically applied load.

Concrete and Steel Properties—Two grades of reinforcing steel, Grade 40 and Grade 60, are commonly used in reinforced concrete construction. For primary structural members, such as beams and columns, Grade 60 reinforcing steel is the more common in the United States. Because Grade 40 steel usually has greater ultimate strain capacity and delayed strain hardening, the deformation capacity of structural members using Grade 40 reinforcing steel is less critical than that of otherwise equivalent members using Grade 60 steel. In the current study, Grade 60 steel is assumed for both longitudinal and transverse reinforcement. A typical stress–strain (σ–ε) curve of Grade 60 steel is shown in Fig. 3.2a. The following properties are assumed:

1. Reinforcing steel has a modulus of elasticity (E) of 29000 ksi and a yield stress (f_y) of 69 ksi, which is 15% higher than the specified nominal yield stress.

2. Strain hardening behavior starts at the strain three times the yield strain, with a strain hardening stiffness (E_{sh}) of 1500 ksi. The ultimate strength is 110 ksi, which occurs at the strain of 0.09. When a strain value of 0.12 is reached, the steel is considered fractured.

3. The stress–strain relation is linear before yield, maintains the yield stress from the onset of yield to the onset of strain hardening, and follows a quadratic curve [67] thereafter until fracture.
Concrete having a nominal compressive strength \( f' \) of 4000 psi is assumed. The maximum stress is assumed to occur at the strain of \( \varepsilon_0 = 0.002 \). Confinement due to transverse reinforcement in the form of rectangular hoops is considered, and the stress-strain relations for unconfined and confined concrete in the form proposed by Park, et al. [42] is used. According to this model, the descending slope of the stress-strain relation is a function of the center to center spacing \( s \) of the transverse steel. In defining the descending slope, a fixed spacing of four inches is always assumed, and the variation of confinement is considered by varying the transverse steel ratio \( \rho_s \). Typical stress-strain relations for unconfined and confined concrete so defined are shown in Fig. 3.2b.

Dimensions of Beams and Columns—The aspect ratio and dimensions of beams and columns may vary over a wide range, and it is difficult to establish any bound for their variations except those specified by the design code. In order to minimize the number of variables in this parametric study, the effects of member sizes on the deformation capacities of reinforced concrete members are not considered, and only one beam section of 12 in. by 24 in. and one column section of 20 in. by 20 in. are considered. Other sections should be investigated in order to generalize finding of the study.

Longitudinal and Transverse Reinforcements in Beams—Current codes specify upper and lower bounds of the longitudinal steel ratio \( \rho \) of a beam so as to avoid non-ductile failure modes [2,3]. For seismic design, the maximum allowed longitudinal reinforcement ratio according to the ACI Code is 2.5%, and the minimum allowed steel ratio is approximately 0.33% for Grade 60 steel. In common practice, most beams have a maximum steel ratio in the range from 2/3% to 1.5%. Beams having steel ratio ranging from 0.0067 to 0.025 are studied in the current program.

For beams the ratio of the minimum to maximum longitudinal reinforcement \( \rho' / \rho \) must be at least 0.5 according to seismic provisions of the ACI Code [2]. When calculating the steel ratios \( \rho = A_s / bd \) and \( \rho' = A_{s'} / bd \), a rectangular section or the web in a T-beam section is considered. In cases that floor slabs are cast monolithically with the beam, part of the slab reinforcement is effective in tension in resisting bending moment acting on the
beam [9,13,14,21,22]. Furthermore, there is usually a larger amount of steel on the top of a beam than on the bottom. When part of the slab steel is included, the effective ratio of minimum to maximum steel may be less than the specified value of 0.5. For this reason, this ratio is permitted to vary from 0.25 to 1.0 in the current study.

Transverse reinforcement in beams may be determined by shear strength requirements and by detailing requirements. To cover most of the practical situations, volumetric transverse steel ratios ($\rho_t$) in the range from 0.5% to 2.0% are considered in the current study.

Longitudinal and Transverse Reinforcements in Columns—Columns usually are reinforced with equal amounts of longitudinal steel on opposite sides in typical frame construction. The total steel ratio ($\rho_t$) allowed by the design code is in the range from 1% to 6% for regions of high seismicity. Practically, the total steel ratio falls in the range between 1% and 4%. For the purpose of estimating the deformation capacity of columns in the current study, a single steel ratio of $\rho_t = 3.0\%$ is used as a representative value. Considering a column section under axial load on the order of the balanced load [43], as shown in Fig. 3.4, when the reinforcing steel on one side of the column yields in tension, the steel on the other side is also likely to yield, but in compression. Thus, neglecting strain hardening, the forces developed in the reinforcing steel tend to cancel each other, and the location of the neutral axis is determined primarily by the amount of axial load acting on the column. Because the tension and compression steel forces tend to cancel, the steel ratio is theoretically not a significant variable under the assumed axial load. Thus, the restriction that $\rho_t = 3.0\%$ in the current study does not severely limit the usefulness of any conclusion.

A minimum amount of transverse steel is required by current codes [2,3,63] to provide confinement so that the axial load carrying capacity of a column can be maintained after the spalling of the concrete cover. For 4000 psi concrete, Grade 60 steel, and a square column section, the minimum amount required by the ACI Code is equivalent to a transverse steel ratio of 1.2%. By satisfying all other detailing requirements in the ACI Code, the actual transverse steel ratio may become much higher. In the current study, transverse steel ratios ranging from 0.8% to 2.8% are considered.
Axial Loads Acting on Columns—The intensity of axial load acting on a column can be expressed as the axial load ratio, \( P/A_g f'_c \). As illustrated by the data of Fig. 2.1, column behaviors strongly influenced by the level of axial load. A range of axial load ratios from 0.1 to 0.6 is considered here to account for most of the practical situations.

Length of Plastic Hinges—The plastic hinge length, \( l_p \), is defined by Eq. (c). The value \( Z \) varies due to different loading conditions and to different clear span to effective depth \( (L_n/d) \) ratios. In typical beams under combined gravity and lateral loads, the inflection point is at approximately 1/3 of the span length from the face of the column. Assuming a span to depth ratio of \( L_n/d = 10 \), then the value of \( Z \) can be estimated as \( Z = 3d \). For columns in the bottom story, where the deformation demand is usually the highest, the value of \( Z \) usually falls in the range from 1/2 to 2/3 of the clear story height \( (H_n) \) from the bottom, depending on the relative stiffness of beams and columns. Considering \( H_n/d \) varies from 5 to 6, a value of \( Z = 3d \) represents cases in which the inflection points fall into the above range. Therefore, a constant value of \( Z = 3d \) is used for all beams and columns, which results in a constant plastic hinge length of 0.65\( d \).

A summary of each parameter discussed and the range of variation is shown in Table 3.3. Examples of typical reinforced concrete beam and column sections in earthquake-resistant frame construction can be found in Fig. 3.5.

3.3.2 Results of the Parameter Study

Calculated beam plastic hinge rotations are plotted versus the ratio of minimum to maximum longitudinal steel \( (\rho'/\rho) \) and the transverse steel ratio \( (\rho_s) \) in Fig. 3.6 (There is a series of plots, Fig. 3.6a–3.6f, each corresponding to a fixed value of maximum longitudinal steel ratio, \( \rho_c \)). For columns, the plastic hinge rotations are plotted versus the axial load ratio \( (P/A_g f'_c) \) and the transverse steel ratio \( (\rho_s) \), as shown in Fig. 3.7. Only results for one longitudinal steel ratio are presented for columns for the reason described previously.

Several observations of a general nature can be made from the data plotted in Fig. 3.6 and 3.7. For beams, the calculated plastic hinge rotation capacities increase with increasing
transverse steel ratio, increase with increasing ratio of minimum to maximum longitudinal steel ratio, and increase with decreasing longitudinal steel ratio. For columns, the calculated plastic hinge rotation capacities increase with increasing transverse steel ratio and increase with decreasing axial load. These findings have been borne out in part by experimental studies (Ch. 2).

The magnitudes of the rotation capacities for typical configurations are of particular interest in the present study. A typical beam may be considered to be one having longitudinal steel ratio not exceeding 1.5 percent, ratio of minimum to maximum longitudinal steel ratio not less than 0.5, and transverse reinforcement ratio not less than 0.005. For this typical beam, the calculated plastic hinge rotation capacity is approximately 0.02. Assuming that the calculated rotation capacity is a fair measure of actual capacity, a well-proportioned frame composed of typical beams would be expected to have a drift capacity of at least 0.02. A typical column may be considered to be one having transverse reinforcement ratio of at least 0.012. The calculated plastic hinge rotation capacity for this minimum transverse steel ratio can be as high as 0.02 for low axial loads, or as low as 0.01 for high axial loads.
CHAPTER 4

DISPLACEMENT RESPONSE
OF IDEALIZED SDOF SYSTEMS

Experimental studies in the laboratory (Ch. 2) have indicated that reasonably well
detailed reinforced concrete frame components can undergo deformations corresponding to
interstory drifts well in excess of two percent of interstory height. These findings are sup­
ported by analytical studies of plastic hinge rotation capacities (Ch. 3), which indicate
similar drift capacities for typical frame elements. The analytical studies also provide guid­
ance on details and proportions required to ensure the achievement of a minimum drift
capacity. If techniques can be developed for the calculation of maximum drifts during
earthquakes, structural evaluation or design could be achieved on the basis of drift alone.

Development of these techniques is the subject of the remaining chapters of this study. The
subject of the present chapter is the development of practical means of estimating the peak
values of inelastic displacement response of single-degree-of-freedom (SDOF) systems under
the action of earthquake induced ground accelerations.

4.1 Review of Previous Studies

Over the last 30 years an enormous effort has been devoted to understanding struc­
tural response under dynamic excitations. The development of elastic as well as inelastic
acceleration response spectra [20,24,36,37] has formed the basis of current design criteria
for earthquake resistant design of building structures [53,63]. Some of the studies related
to the current study are briefly reviewed in the following paragraphs.

The overall response of a building system to dynamic loadings is controlled by the
dynamic characteristics of the loading and of the structural system. The response of linear
elastic systems is particularly sensitive to harmonic loadings because of the effects of res­
onance [4,10,11]. Damping properties of the system, period ratio between the system and
the exciting force and number of load cycles are important factors affecting the magnitude of maximum response of linear elastic systems. On the other hand, the response of inelastic systems is often found to be controlled by large and long duration load impulses. During a long impulsive loading, the existence of applied loads after yielding of a system tends to force the system to undergo a large inelastic displacement. In contrast to elastic systems, yielding of a system changes the apparent period and keeps the system from resonance during harmonic loadings \[11,35\].

Studies have shown (Newmark, et al.) \[35,36,37\] that for each earthquake there exist different period ranges in the linear elastic response spectra of SDOF systems in which roughly constant response quantities are preserved (Fig. 4.1). In those period ranges of constant velocity and displacement response, it is further found that maximum inelastic displacement is approximately the same as the elastic displacement if the elastic displacement spectrum is represented by a smoothed curve. These findings were based on statistical studies of a large number of samples of different earthquake ground motions recorded from earthquakes having different intensities and at varying epicentral distances. Different hysteretic models (ranging in complexity from simple elasto-perfectly plastic models to sophisticated stiffness degrading models), and different damping ratios (from 2% to 20% of the critical value) were considered in the statistical study.

The observation that inelastic and elastic displacements are the same for a particular period range has significant design implications. First, it provides a simple means of estimating drift during an earthquake. Second, if drift and damage are interrelated, then the observation suggests that it is the initial stiffness of the structure that controls damage. Strength is largely unimportant.

Similar observations have been made by other researchers (Shimazaki and Sozen) using different approaches \[58,59\]. In those studies, the energy response of structures to earthquake ground motions was studied in addition to the displacement response. From a large number of calculated responses, it was found that when the periods of structures were longer than a characteristic value, the spectral value of input energy was constant or diminished.
slightly, regardless of the strength of the system. Consequently, the input energy spectrum was idealized as a bi-linear curve and the characteristic period $T_g$ of each ground motion was identified as the period corresponding to the break point on the idealized bi-linear energy spectrum. It was recognized that this characteristic period corresponds approximately to the period at the intersection of the constant velocity response and constant acceleration response regions (Fig. 4.1).

As observed in Shimazaki and Sozen’s study, when the period of a structure was longer than the characteristic period ($T > T_g$), the maximum inelastic displacement response, no matter how strong the structure was, tended to be reasonably bounded by the elastic displacement response spectrum. The observation was explained qualitatively using the concept of input energy. Yielding of a system results in an increase in the effective period. If the initial period $T$ is longer than the characteristic period $T_g$, the increase in period due to yielding will not produce a corresponding increase in input energy. Thus, there is no tendency to develop larger displacement than the elastic response displacement in order to absorb the input energy. However, if the system has an initial period shorter that $T_g$, an increase of effective period will cause increase of input energy, and larger displacement will result.

Shimazaki and Sozen also observed that inelastic displacements were equal to or less than the elastic values for periods $T$ less than $T_g$ under certain conditions, as follows. They defined a strength ratio $C_y/S_a$, in which $C_y$ is the ratio of base shear to total weight and $S_a$ is the elastic response acceleration expressed as a fraction of the gravity acceleration. If the sum $T/T_g + C_y/S_a$ exceeds unity, then the inelastic displacement was found to be equal to or less than the elastic value.

Shimazaki and Sozen did not define the expected inelastic displacement response for systems having the sum $T/T_g + C_y/S_a$ less than unity. If a general displacement design method is to be developed, it is necessary to define the displacement response for all period ranges. The pursuit of such a definition is the main object of the study summarized in the following sections.
4.2 Estimate of Maximum Inelastic Displacement

The problem of estimating maximum displacement response (either elastic or inelastic response) of a SDOF system is considered in two period ranges divided by the period $T_g$ in the current study. It has been established [37, 59] that for structures having initial periods $T > T_g$, the maximum inelastic displacement can be estimated as being equal to the peak value of elastic displacement having the same period $T$. This finding is convenient because it enables displacements of buildings in this period range to be estimated readily using established elastic analysis techniques.

In the period range of $T < T_g$, it is understood that the maximum inelastic response of a system is affected by (1) the characteristics of the exciting earthquake, (2) the initial period $T$, and (3) the yielding strength of the system. However, it has been difficult to establish any quantitative relation between the above three factors and the peak displacement response of inelastic systems. In order to establish an estimate of the peak value of inelastic displacement response in this period range, the following assumptions were made.

(a) The maximum inelastic displacement response of a structure is controlled by its stiffness and strength, and by the largest earthquake acceleration impulse. The response can be characterized by an inelastic system subjected to impulsive loadings. The earthquake induced impulse loading can be idealized as having a triangular distribution with time, a peak intensity of $MA_{g,\text{max}}$ at time $td/2$ and a duration of $td$, where $M$ and $A_{g,\text{max}}$ represent the mass of a SDOF system and the peak ground acceleration, respectively.

(b) The hysteretic behavior of a SDOF system can be simplified as being elasto-perfectly plastic.

(c) A structures has so called zero initial conditions when the load pulse starts to act on it.

(d) The duration of the acceleration pulse is generally short (One of the longest single acceleration pulses observed is in the accelerogram derived at Pacoima Dam, and has a duration of approximately half of a second). The maximum displacement response of structures
is reached in a time on the same order of the load duration when subjected to impulsive loadings, before the damping mechanism can absorb much energy from the structure. For this reason only the undamped response is considered in this part of the study.

Having the state of the system defined with the above assumptions, undamped shock spectra [4] with a triangular shaped impulse loading and an elasto-plastic resistance function can be used to study the response characteristics.

Undamped shock spectra are shown on a log–log scaled plot (Fig. 4.2). The ordinate of the plot is the ductility ratio \( \mu \), defined as \( \Delta_{\text{max}}/\Delta_y \), and the abscissa is the ratio of applied load duration to the initial period of the system \( t_d/T \). It is observed that for each fixed strength ratio \( \eta = R_y/F_{\text{max}} = R_y/MA_{g,\text{max}} \) (Note that \( R_y \) and \( F_{\text{max}} \) are designated as \( R_m \) and \( F_1 \), respectively, in Fig. 4.2) that is equal or less than 0.8, the relation between \( \mu \) and \( t_d/T \) is approximately linear on a log–log scale, but with different slopes for different values of \( \eta \). This linear relation is approximately valid up to a load duration to structure period ratio \( (t_d/T) \) of 5, based on the available data (Fig. 4.2). Considering design earthquakes having a peak acceleration on the order of 0.5g, strength ratios between 0.2 to 0.8 would correspond to yielding strength of 10 to 40 percent of the weight of the system, which is in the practical range of interest.

Analytically, this linear relation on a log–log plot can be expressed as

\[
\log(\mu) = a\log(t_d/T) + \log(b)
\]

(4.1)

where \( a \) and \( b \) are constants depending only on the strength coefficient \( \eta \) of the structure. Geometrically, the quantity \( a \) represents the average slope of each line of constant \( \eta \) values, and can be approximately expressed as

\[
a \approx -1.6\eta + 2
\]

(4.2)

Equation (4.1) can be used to derive a relation between maximum inelastic displacements for \( T < T_g \) and the elastic displacement at \( T_g \), as follows:
Denote
\[ \alpha = \frac{T}{T_g} \quad (4.3) \]
and
\[ \beta = \frac{t_d}{T_g} \quad (4.4) \]
then,
\[ \frac{t_d}{T} = \frac{t_d}{T_g} \times \frac{T_g}{T} = \frac{\beta}{\alpha} \quad (4.5) \]
Rewrite Eq. (4.1) as
\[ \mu = b\left(\frac{t_d}{T}\right)^\alpha \quad (4.1a) \]
and for the same load duration \( t_d \) and strength coefficient \( \eta \), the ductility factors for \( T \) and \( T_g \) can be expressed as Eq. (4.6a) and (4.6b), respectively, by substituting Eq. (4.4) and (4.5) into Eq. (4.1a),
\[ \mu_T = b\left(\frac{t_d}{T}\right)^\alpha = b\left(\frac{\beta}{\alpha}\right)^\alpha \quad (4.6a) \]
\[ \mu_{T_g} = b\left(\frac{t_d}{T_g}\right)^\alpha = b\left(\frac{\beta}{\alpha}\right)^\alpha \quad (4.6b) \]
Dividing Eq. (4.6b) by Eq. (4.6a) results in
\[ \frac{\mu_T}{\mu_{T_g}} = \frac{b\left(\frac{\beta}{\alpha}\right)^\alpha}{b\left(\frac{\beta}{\alpha}\right)^\alpha} = \left(\frac{1}{\alpha}\right)^\alpha \quad (4.7) \]
By definition of the ductility factor,
\[ \Delta_{T,\text{max}} = \mu_T \times \Delta_{T,\text{y}} \quad (4.8a) \]
\[ \Delta_{T_g,\text{max}} = \mu_{T_g} \times \Delta_{T_g,\text{y}} \quad (4.8b) \]
Dividing Eq. (4.8a) by (4.8b), and noting that in the range of constant acceleration response,
\[ \frac{\Delta_{T,\text{y}}}{\Delta_{T_g,\text{y}}} = (T/T_g)^2 \]
results in Eq. (4.9)
\[ \frac{\Delta_{T,\text{max}}}{\Delta_{T_g,\text{max}}} = \left(\frac{T}{T_g}\right)^{2-a} \quad (4.9) \]
Finally, substituting Eq. (4.2) into Eq. (4.9), and using the maximum elastic displacement instead of the inelastic displacement for the period of \( T = T_g \) as the reference value, results in

\[
\frac{\Delta T_{\text{max}}}{\Delta T_{*,\text{elas}}} = \left( \frac{T}{T_g} \right)^{1.6\eta}
\]

or

\[
\Delta T_{\text{max}} = \left( \frac{T}{T_g} \right)^{1.6\eta} \times \Delta T_{*,\text{elas}}
\]

In summary, for a given earthquake, if the elastic displacement response spectrum for a certain damping ratio is obtained by elastic analysis, in the period range of \( T \geq T_g \), the maximum inelastic displacement response of a structure can be estimated as being equal to the elastic spectral displacement. In the period range of \( T < T_g \), the maximum inelastic displacement response of a structure can be estimated using Eq. (4.10) or (4.10a) and the elastic spectral displacement at \( T = T_g \). Therefore, the maximum displacement response for any system, elastic or inelastic, having any period, \( T \), can be estimated based on elastic analysis.

4.3 Analytical Verification of Estimated Displacement Response

4.3.1 Earthquakes Used for Verification

Because several assumptions were made in deriving the relation between elastic and inelastic displacements, it is necessary to verify the relation using recorded earthquake motions. Nine earthquake records, with values of \( T_g \) ranging from 0.35 sec to 1.50 sec, were used as earthquake input. For each recorded earthquake, the value of \( T_g \) was determined from the plot of input energy spectrum (Fig. 4.3–4.11), where the input energy was expressed as an equivalent velocity defined as \( \sqrt{E/M} \). The characteristic period \( T_g \) was defined in the present study as the period corresponding to the first significant peak in the energy response spectrum. This definition is slightly different from that given by Shimazaki and Sozen (as the knee point on the idealized bi-linear energy spectrum). However, the values of \( T_g \) defined herein coincide closely with those defined in Shimazaki’s study. The period
$T_g$ is also marked on the pseudo velocity response spectrum (Fig. 4.3–4.11). Descriptions of each earthquake record used including the peak acceleration and the characteristic period $T_g$ are listed in Table 4.1.

For the inelastic displacement study, all the earthquake records in Table 4.1 were scaled to have a peak acceleration of $0.5g$. With the exception of the two records from the 1985 Chile earthquake and the record derived at Pacoima Dam in the 1971 San Fernando earthquake, the first 20 seconds of the records were used to perform the response calculation. The two Chilean earthquake records have extremely long duration and the peak acceleration occurs in the range between 30 to 40 second. Thus, approximately the first 70 seconds were used to calculate the response. Only 15 seconds of the derived Pacoima Dam record were used because the strong motion in that record ended after approximately 11 seconds.

4.3.2 Structural Models Used for Verification

Responses of SDOF systems having initial periods ranging from 0.05 sec to 3.0 sec were computed for each earthquake excitation. Strength coefficients, $\eta$, of each SDOF system were selected to be 0.2, 0.4 and 0.6, corresponding to yield strengths of 10, 20 and 30 percent of the weight for earthquakes having peak accelerations of $0.5g$.

Two of hysteretic models, bi-linear and stiffness degrading, were used. Also, systems without strain hardening and with strain hardening stiffness equal to 10 percent of the initial stiffness were used for each hysteretic model.

A damping ratio of 5 percent of the critical value was used for all response calculations, as it was believed to be appropriate for large amplitude response under the excitation of design level earthquakes.

Responses were calculated by a micro computer version of NONSPEC, a program to construct inelastic response spectra for SDOF systems [24].

4.3.3 Comparison between Estimated and Calculated Displacements

For each earthquake ground motion, elastic response spectra were created. For each $\eta$ value of 0.2, 0.4 and 0.6, inelastic displacement responses for systems having simple bi-linear
or stiffness degrading behavior, and with or without strain hardening effect, were calculated. The response maxima are plotted in Fig. 4.12-4.20. On these plots, continuous curves represent elastic responses and broken curves represent inelastic responses corresponding to different hysteretic models. In the period range of $T > T_g$, the inelastic displacement response is expected to be bounded by the elastic response. In the period range of $T < T_g$, the expected maximum displacement responses using Eq. (4.10) are shown as continuous curves with centered symbols.

Generally speaking, in the period range of $T > T_g$, elastic displacement response curves provide a fair bound to inelastic displacements, no matter which strength coefficient and hysteretic model is used. This observation is consistent with the results of previous studies [37,59].

It is apparent in Fig. 4.12-4.20 that for $T < T_g$, the maximum inelastic displacement response varies with the strength, with larger values of $\eta$ tending to result in smaller inelastic displacements. In addition, systems modeled with different kinds of hysteretic behavior tend to have different inelastic displacements. In particular, systems having strain hardening tend to have smaller inelastic displacements than equivalent systems without strain hardening. It is noteworthy that the estimated peak inelastic displacements as given by Eq. (4.10), and represented by continuous curves with centered symbols in Fig. 4.12-4.20, provide a reasonable upper bound for most of the calculated responses.

4.4 Discussion

Structural response to earthquake ground motions is a complex process involving many variables. Simplified analysis of the response of structures subjected to idealized loadings, as performed in the development of the shock spectra (Fig. 4.2), can provide useful information about the characteristics of structural response. However, such simplified analyses should not be expected to provide accurate results under all conditions. The following points deserve further discussion.

(1) Comparing the estimated displacement with actual calculated values, it is observed
that the estimated displacements provide a reasonable upper bound for the maximum value of inelastic displacement. In most cases, the ratio of actual calculated displacements to the estimated values falls in the range of 0.5 to 1.0.

(2) Some earthquakes have double peaks of approximately equal amplitude in the energy spectrum (Fig. 4.5 and 4.10). For relatively weak structures ($\eta = 0.2$), it was observed in the corresponding response comparisons (Fig. 4.14 and 4.19) that the calculated inelastic displacements exceed the value estimated from Eq. (4.10a). A plausible explanation for this discrepancy is that as the system yielded, the period extended into the range corresponding to the second peak of the energy spectrum (Fig. 4.5 and 4.10), and the response was thus driven to higher levels than expected. If the characteristic period is defined relative to the second peak, conservative estimates are obtained (Fig. 4.21 and 4.22).

(3) The estimated inelastic displacement in the period range of $T < T_g$ was based on the shock spectra for a single impulsive load. During earthquake ground motions, it is not uncommon that another acceleration pulse in the opposite direction follows immediately after the first one. In this case, the structure may be forced to move in the opposite direction before the maximum displacement due to the first pulse can be developed. Figure 4.23 shows the record of ground acceleration derived at Pacoima Dam and the resulting displacement response of a SDOF system having $T = 0.1$ sec, $\eta = 0.2$ and $\xi = 0.05$. In the range of $t \approx 2.5$ to $t \approx 3.5$ sec (designated as zone “a”), there is a pair of acceleration pulses in opposite directions. The second pulse tends to push the system backwards to the undeformed position before the maximum displacement that could result from the first peak is developed. If the earthquake had stopped just after the first pulse, the calculated displacement response would have followed the dotted line and a larger displacement would have resulted.

Another possible type of ground motion can be found in Fig. 4.23 as in zone “b”, where there are consecutive large acceleration pulses in the same direction (the reverse pulses in the opposite direction in between are too small to change the response significantly). In the case that each of those pulses in the same direction is large enough to cause significant
inelastic displacement to the system, a series of this kind of pulse will result in large accumulated inelastic displacement towards one direction (Fig. 4.23), and it is possible that this displacement will be larger than the displacement resulting from the largest single pulse. It is believed that most of the cases having under-estimated inelastic displacement can be attributed to the effect of this kind of ground motion.

(4) Strain hardening behavior tends to reduce the maximum response, as can be observed in Fig. 4.12–4.22. If the load impulse is viewed as the input of a certain amount of momentum to be converted to hysteretic energy dissipated by the system, it is clear that any increase of resistance due to strain hardening will develop a smaller peak displacement in order to absorb the same amount of energy as a system without strain hardening (Fig. 24). The calculated results show that the estimated displacement (by Eq. (4.10a)) tends to exceed the value calculated for systems having strain hardening.

(5) If a system having stiffness degradation undergoes inelastic response, its stiffness during subsequent response will, of course, reduce (Fig. 4.25) and therefore, the apparent period ratio $T/T_s$ becomes larger. If subjected to another large pulse, the displacement may tend to increase beyond that expected based on the initial period ratio. Thus, stiffness degrading models may tend to have larger inelastic displacement than equivalent bi-linear models. However, this effect was not predominant within the range of data studied.
CHAPTER 5

DISPLACEMENT RESPONSE OF MULTI-STORY FRAMES

The previous chapter established that the displacement response of an idealized SDOF system under earthquake action could be readily bounded over the entire period range by means of elastic analysis. The purpose of this chapter is to extend the study of SDOF systems to multi-degree-of-freedom (MDOF) systems. The study begins by developing equivalent SDOF models for several multi-story frames. The frames are analyzed to draw conclusions as to the efficacy of the SDOF models in representing the complete structure response. Simplified techniques for estimating maximum inter-story drift are developed based on findings of the study.

5.1 Representation of Multi-Story Frames by Equivalent SDOF Systems

5.1.1 Development of Equivalent SDOF Systems

In order to apply the technique of estimating the maximum displacement of SDOF systems to the analysis and design of multi-story reinforced concrete frames, the multi-story frames need first to be converted to equivalent SDOF systems that represent the general response characteristics of those frames.

The development of equivalent SDOF systems can be traced to the early 1960s [4]. Some later work can also be found in the studies by Saiidi and Sozen [49,50]. Available equivalent SDOF models attempt to model both the displacement and the resistance of multi-story buildings. Consequently, some assumptions have been necessary to define the equivalent resistance function. These assumptions usually require inconsistencies in the mathematical derivation of the equivalent SDOF model. In the course of this study, a modified version of equivalent SDOF model was developed (App. B) and used throughout the presentation in the rest of this chapter.

The model developed in App. B uses the concept of generalized coordinates. A de-
flection shape corresponding to that which occurs at approximately 1% average drift index (ADI), defined as the displacement at the roof level divided by the total height of the structure, is used as the assumed constant shape vector. The equivalent yielding resistance is defined as the dot product of the shape vector and the load vector that cause the “yielding” of the structure. Details of the derivation of the equivalent SDOF model and definitions of those equivalent quantities can be found in App. B.

5.1.2 General Descriptions of Frames Used in the Study

Five- and ten-story generic frames having elastic fundamental periods equal to 0.57 and 1.19 second, respectively, were used in this study. The frames were assumed to have uniform story height of 10 ft and two equal spans of 20 ft (Fig. 5.1). For the sake of simplicity, the earthquake ground motions and the structural response were restrained within the plane of the frame.

The mathematical model of the frames consists of available beam-column elements that are incorporated into a general purpose nonlinear analysis program [47]. Centerlines were used to represent the location of structural elements. Beams and columns were assumed to have constant cross sections over the height of the building. Consequently, the flexural stiffness for beams, and for interior and exterior columns, were assumed to have constant values over the entire height.

At the bottom of each column (at the foundation level), a fixed boundary condition was assumed. Therefore, the interaction between the superstructure and soil foundation was totally ignored. In addition, at the ends of beams and columns, rigid offsets were assumed to account for relatively stiff joint zones. Mass and stiffness proportional damping equal to five percent of the critical value for the first two modes were used.

Beam-column elements were modeled as having a simple bi-linear moment-rotation relation. In the study of SDOF systems, the degradation of stiffness under load reversals did not show significant effect on the displacement response. Thus, no stiffness degrading was considered in order to minimize the computer time for conducting inelastic frame analyses
under dynamic loadings. The strain hardening ratio for the bi-linear moment-rotation model was arbitrarily assumed to be five percent of the initial stiffness. No moment-axial load interaction was considered for columns.

Three different kinds of strength distribution between beams and columns and over the height of the structure were considered, designated as Type I, II and III (Fig. 5.2). (Reasons for choosing these three kinds of distribution will be discussed later together with the analysis of drift distribution.) Type I consists of beams and columns having the same pattern of strength variation. The flexural strength of beams at the roof level is set as the reference value and denoted as 1.0 (Fig. 5.2). The negative moment strength for the roof level beams is determined as approximately the fixed end moment under factored gravity loads, assuming nominal dead load of 150 psf and live load of 50 psf. The strength for positive moment is taken to be half of the negative strength. Within the lower four tenths of the height, the beam moment capacities are uniform and are 2.5 times the values in the top level. Strengths vary linearly in between. Column strengths are determined so that a specified column-beam strength ratio \( R = \Sigma M_{\text{column}}/\Sigma M_{\text{beam}} \) is obtained at each beam-column joint, except at the roof level. Strength distribution Type II is defined to have the same column strength as in Type I, but the strength of the beams is constant over the height. Both beams and columns are assigned uniform strength over the entire height in Type III.

5.1.3 Representation of Frames by Equivalent SDOF Systems

Each frame, having different strength distributions and column-beam strength ratios, was analyzed statically under the action of monotonically increasing lateral loads, as described in App. B. Load-deformation curves (Fig. 5.3–5.4) as well as normalized deflected shapes at 1% of ADI (Fig. 5.5–5.6) for those frames considered were obtained by static analysis. Then, equivalent quantities were determined using formulas presented in App. B. The results from the static analysis and the values of equivalent quantities for five- and ten-story frames are summarized in Tables 5.1 and 5.2, respectively.
5.2 Estimation of the Peak Roof Level Displacement

Earthquakes having different characteristic periods were used to analytically excite the five- and ten-story frames. The ground acceleration records for each earthquake were scaled such that for an elastic SDOF system having its period equal to the fundamental period \( T_1 \) of a frame, a lateral displacement corresponding to 1% of the height of the frame would result (assuming a fundamental period of \( T_1 = N/10 \), five percent damping, and 10 ft story height).

Based on the properties of the equivalent SDOF system defined in the previous section, the peak value of roof displacement of each frame can be estimated using the elastic displacement response spectrum and techniques described in App. B. In some cases, inelastic analysis of the equivalent SDOF system was conducted as a reference. A general purpose nonlinear analysis program, ANSR-I [33], was employed to conduct the numerical analysis of multi-story frames. The comparison of peak displacement responses to the estimated values is of particular interest in this part of the study.

Based on the results of the SDOF system study in Ch. 4, it is expected that for structures having \( T_1 > T_g \) and \( T_1 < T_g \), the strength of the frame is going to play different roles in affecting displacement response of the structure. Therefore, the results from this study are presented in two parts accordingly.

5.2.1 Roof Level Displacement for Structures Having \( T_1 > T_g \)

Tables 5.3 and 5.4 summarize the results for five- and ten-story frames \( (T_1 = 0.57 \text{ sec and } 1.19 \text{ sec, respectively}) \) under the excitation of earthquakes having \( T_g \) such that \( T_1 > T_g \). The first two columns in the table list the earthquakes and types of resistance for each frame analyzed. The third column is the equivalent strength ratio, \( \eta^* = R_g^*/M^*A_{g,max} \), as described in App. B. Following are the roof displacement values (calculated and estimated) and finally, the ratios of calculated and estimated displacement values.

It is shown in Tables 5.3 and 5.4 that even though the strength ratios are different from each other for different types of strength distribution, the calculated maximum displacement
responses are approximately the same for a given earthquake. This result is expected from conclusions of Ch. 4 that indicate maximum displacement response is effectively independent of strength if \( T_1 > T_g \). Figures 5.7 and 5.8 show typical displacement response histories of the roof, second and third stories. Although the waveforms are not quite the same, the peak displacement responses (absolute values) at the roof level are approximately the same. The differences of the peak displacement values for lower stories (where maximum inter-story drifts tend to occur) are usually more noticeable than for the roof level. This result will be discussed in the following section along with the discussions on drift distribution.

Generally, response estimates based on the elastic displacement response spectrum tended to exceed the maximum roof displacement calculated by inelastic response history analysis (see the last column of Tables 5.3 and 5.4). In some cases, the estimated values are twice as large as the calculated values. However, the calculated displacements from inelastic frame analysis and that from inelastic equivalent SDOF system analysis (where available) are close. Therefore, the discrepancy is most likely caused by the difference of elastic and inelastic displacement for SDOF systems. In the statistical study reported by Newmark and Riddell [37], it was found statistically that inelastic systems tended to displace less than elastic systems in the period ranges of constant velocity and displacement response. The range of difference between elastic and inelastic response maxima is also within that identified in Ch. 4 of this study (see Fig. 4.12–4.20).

### 5.2.2 Roof Level Displacement for Structures Having \( T_1 < T_g \)

Only five-story frames (\( T_1 = 0.57 \) sec) were subjected to ground motion records (1940 El Centro, 1978 Miyagi and 1971 Santa Barbara records) such that \( T_1 < T_g \). In order to see the effect of the total strength, frames having strengths twice the original values (designated as Type Ia, IIa and IIIa) were also analyzed subjected to the Miyagi record. Table 5.5 summarizes the results in the same format as used in Tables 5.3 and 5.4.

Similar to the results for cases of \( T_1 > T_g \), roof level displacement responses obtained from the analysis of the equivalent SDOF systems agree fairly well with those obtained from
the analysis of multi-story frames. Because the estimated maximum inelastic displacement is based on the elastic displacement at the period of $T_2$, the difference of elastic and inelastic displacements at that period (for cases considered here, inelastic displacements were usually smaller than the elastic value) tends to propagate towards the estimate of maximum inelastic displacement for structures having $T_1 < T_2$. For this reason, calculated maximum roof displacements for MDOF systems tended to be less than the estimated values.

In contrast to observations for structures having $T_1 > T_2$, the total strength ratio ($\eta^*$) has notable effects on the maximum inelastic displacement response (Table 5.5). For the cases studied, structures having approximately the same strength ratios (eg., Miyagi earthquake, strength Type I versus Type IIa) result in roughly the same peak inelastic displacement, regardless of the type of strength distribution. Structures having different strength ratios develop different maximum roof displacements. These observations are consistent with those anticipated based on the study of SDOF systems in Ch. 4. Typical displacement response histories for frames having different strengths can be observed in Fig. 5.9.

The available results (Tables 5.3-5.5) indicate that calculated peak roof displacements tend to fall into the range of 50% to 100% of the values estimated based on the elastic displacement response of SDOF systems. It is concluded that the techniques for estimating the maximum displacement response provide a reasonable upper bound of maximum roof displacements for multi-story structures. For structures having $T_1 > T_2$, different types of strength distribution and different values of total strength ratio tend to result in approximately equal roof level displacement, and that displacement is approximately equal to the elastic displacement. For structures having $T_1 < T_2$, the maximum roof displacements are affected by the strength of the structure. Given the equivalent strength ratio, $\eta^*$, as defined in App. B, the maximum inelastic roof displacements can be reasonably bounded for structures having $T_1 < T_2$ using Eq. (4.10).
5.3 Estimation of Maximum Inter-Story Drift Index

The value of maximum roof displacement, or of the ADI, is a direct and efficient way to quantify the overall displacement response of a building. However, the value of the ADI provides no direct information about localized deformation within a structure. If the value of the story drift index (SDI), defined as the ratio of the inter-story lateral displacement to the inter-story height, for each story is the same as the ADI, the structure is said to deform uniformly. On the other hand, if in some stories the value of the SDI is much larger than the ADI, concentrated local damages may occur in those stories. A coefficient of distortion (COD) can be defined as the ratio of the maximum value of the SDI to the ADI. To provide a picture of maximum lateral displacement and how it is distributed over the height of a building, a pair comprising the ADI and the COD are used in the following.

The value of the COD is almost always greater than unity, even if the response is limited to the elastic range. In the case of inelastic response, a larger value of the COD is usually expected. In order to control the level of damage of a building under earthquake actions, control of the maximum value of the SDI is desirable. Methods have been developed to estimate the maximum roof displacement of multi-story buildings subjected to given earthquakes as described previously. The analytical responses of reinforced concrete frames subjected to earthquake induced ground acceleration were studied to find a general trend of the COD for structures having continuous distributions of stiffness and strength.

Elastic analysis of multi-story frames reveals that an increase in the higher mode response tends to cause a larger COD for a given roof displacement [11]. Therefore, the frequency content of earthquake ground motion in conjunction with the periods of the building will affect the value of the COD. Other factors that will affect the distribution of the lateral displacement of a building, for a given earthquake, are the stiffness and strength distributions within the structure. The stiffness distribution has been assumed to be uniform along the height of buildings in the current study. The rest of the study in this chapter is devoted mainly to the effects of strength distributions on displacement distributions.
Three types of strength distribution were studied as mentioned before (Fig. 5.2). Current practice of seismic design through a strength design approach results in a strength distribution approximately as represented by Type I (Fig. 5.2). Types II and III (Fig. 5.2) were selected following the idea that if the total strength had little effect on maximum displacement response for structures having $T_1 > T_9$, satisfactory performance may be possible for buildings having strength provided to support gravity loads. Type II was designed such that all beams had the same strength as the roof level beams (as required for gravity loads) and column strength was kept the same as that in Type I. Finally, both beams and columns were assigned constant strength over the height (Type III) to test an extreme case.

Both static analyses under lateral loads at each floor level (distributed over the height in a triangular pattern with the largest load at the top) and dynamic analyses under earthquake excitations were conducted. The static analyses illustrated the effect of strength distribution alone, whereas the dynamic analyses took both the strength distribution and the dynamic features of the earthquake response into account. Typical normalized deflection shapes from static analyses are shown in Fig. 5.10 and 5.11. Normalized deflection shapes and the drift distributions over the height from typical cases of dynamic analyses at the moment of maximum inter-story drift can be found in Fig. 5.12–5.14. A summary of results from this part of the study are listed in Tables 5.6 and 5.7 for five- and ten-story frames, respectively.

As indicated in Tables 5.6 and 5.7, and for both static and dynamic loadings, frames having strength distribution Type I and Type II have practically the same values of the COD. The values of the COD in these situations were also found to be independent of the total strength of buildings. On the other hand, frames having strength distribution Type III incurred consistently larger values of the COD than did the other two types.

The last column of Tables 5.6 and 5.7 contains the ratios of the COD from dynamic and static analyses. The values of the COD from static analyses were determined at 1% ADI. It has been noticed that once the frames are significantly yielded, the deflection shape becomes effectively stable, especially for strength distributions Type I and II, as shown typically in Fig. 5.10 and 5.11. Therefore, the ratios shown in the tables are also valid if more than 1%
ADI is developed. The average ratios of the dynamic COD to the static COD for five-story and ten-story frames are approximately equal to 1.15 and 1.25, respectively, based on the available data. This ratio can be attributed loosely to the effects of “higher modes”. The smaller value for five-story frames may be explained in that higher modes contribute less to the displacement response than to ten-story (or higher) ones. The relatively narrow range of ratios of the CODs from dynamic and static analysis enables the maximum expected value of the COD during earthquake response to be estimated from static analysis.

5.4 Summary

The results and observations obtained in this part of the study can be summarized as follows:

(1) The displacement response at the roof level of a multi-story frame subjected to earthquake ground motions can be approximately represented by an equivalent nonlinear SDOF system constructed as described in App. B.

(2) The peak value of displacement response at the roof level of multi-story frames subjected to earthquake ground motions can be reasonably bounded by the estimated displacement response of SDOF systems using the methods developed in Ch. 4.

(3) The maximum value of the COD of regular frame structures resulting from earthquake excitations can be estimated as the value of the COD from static analysis amplified by a factor of 1.15-1.25. Combining this quantity with the estimated maximum roof level displacement (or ADI), the maximum value of the SDI can be reasonably bounded for given earthquake ground motions and structural systems. In the case that static analysis is not available, based on results presented in this chapter, the maximum value of the COD may be estimated as being in the range from 1.5 to 2.0.

(4) If the inelastic responses of buildings under design level earthquakes are of primary concern, structures having strength distribution Type II can perform as well as those having Type I if the fundamental period of the structure is larger than the characteristic period
of the ground motion. In the event that $T_1 < T_9$, structures having strength distribution Type II need to be upgraded to the same level of strength ratio as Type I in order to achieve competitive behavior. The behavior of structures having strength distribution Type III is not as desirable as behavior obtained with the other two types.
CHAPTER 6
APPLICATION OF DISPLACEMENT ESTIMATION
TO EARTHQUAKE RESISTANT DESIGN

Methods to estimate the peak values of the displacement response of reinforced concrete structures subjected to earthquake ground motions were developed in Ch. 4 and Ch. 5. In this chapter, a design displacement response spectrum (DDRS) will be constructed based on the elastic displacement response spectrum for a given earthquake. In addition, a displacement-based design approach using the DDRS is outlined in comparison with the UBC [63] base shear strength design method. Design examples using strength design and displacement design approaches, and expected behavior resulting from each design method, are presented.

6.1 Construction of Design Displacement Response Spectrum

It has been shown in Ch. 4 that the peak displacement response of SDOF systems can be characterized in two period ranges divided by the characteristic period $T_{g}$, which is earthquake and site dependent. In the period range of $T > T_{g}$, the maximum inelastic displacement responses are approximately equal to the maximum elastic displacement response. Therefore, when a large number of earthquake records of desired intensity are available for a given site condition, a smoothed elastic displacement response spectrum with appropriate values of damping can be constructed on a statistical basis, and used as the design displacement response spectrum. In the period range of $T < T_{g}$, once the spectral displacement at the period $T = T_{g}$ is determined, Eq. (4.10) can be used to construct the design displacement response spectrum with respect to different strength ratios.

In many cases, there do not exist sufficient earthquake records by which to obtain the smoothed design spectrum. Instead, the peak ground motion acceleration, velocity, and displacement can be estimated from localized seismological and geological analysis.
In these situations, elastic response spectra can be constructed using the procedures and amplification factors described by Newmark and Hall [35]. The characteristic period, $T_0$, can be recognized as the period at which the constant velocity and constant acceleration response curves coincide. Once the smoothed elastic response spectrum and the characteristic period are determined, a design displacement response spectrum can be established in the same manner as described in the previous paragraph.

The construction of a design displacement response spectrum faces the same kinds of difficulties and uncertainties as are faced in the construction of a design acceleration response spectrum. Extensive discussion of this aspect is not intended in the current study. A generic design displacement response spectrum is shown in Fig. 6.1. Examples of design displacement response spectra constructed using a single recorded earthquake can be found in Fig. 6.6 and 6.8.

6.2 Basic Steps of Strength Design Method for Earthquake Excitations

The current state of practice for earthquake resistant design of building structures is considered to be represented by the Uniform Building Code [63]. Basic steps to conduct an earthquake resistant design using this method can be briefly summarized as the following.

1) Define the design earthquake in terms of an acceleration response spectrum.

2) Perform a preliminary design of the structure.

3) Define a structural model and determine the properties of the model, such as the total weight $W$ of the building, the fundamental period $T_1$ (commonly, using gross-section properties) and the structural system coefficient $R_w$.

4) Determine the design criteria in terms of importance factor $I$.

5) Determine the minimum design base shear $V$, as

$$V = \frac{ZIC}{R_w}W$$

6) Distribute the base shear over the height of the structure.
6.3 Basic Steps of Displacement Design Method for Earthquake Excitations

A displacement design approach using the concept of displacement control can be performed using steps that parallel the strength design method. The displacement method is appropriate for designing buildings to withstand ultimate level earthquakes for which extensive inelastic response may occur. The design for small magnitude earthquakes usually requires elastic response and thus may be accomplished using forces rather than displacements. Basic steps of the displacement design method are outlined and briefly explained in the rest of this section.

(1) Define the design earthquake in terms of a displacement response spectrum as described in Sec. 6.1.

(2) Perform a preliminary design of the structure.

(3) Define a structural model and determine the properties of the model, such as \(\{\phi\}, T, M^*, L^*, R^*_u\), by a static analysis as described in App. B or by reasonable estimation. The properties should be determined as closely to the real working conditions as possible. It is recommended that cracking of the structural members under service load conditions be considered. (In the strength design approach, an underestimate of the period of a structure usually requires larger design earthquake loads and results in a more “conservative” design. Just opposite to the strength design approach, an un-conservative
design generally results if the period of the structure is underestimated, for example, by using gross-section properties.)

(4) Determine the design criteria in terms of a maximum allowable inter-story drift index (SDI). Then, using an estimated coefficient of distortion (COD, see Ch. 5), an allowable roof displacement is determined.

(5) Modify the design displacement response spectrum (multiplied by the factor of $L^*/M^*$ as discussed in App. B) so that the displacement at the roof level can be read from the modified spectrum (Fig. 6.6 and 6.8).

(6) For a given structure, determine the minimum stiffness required, in terms of the maximum allowable fundamental period $T_{max}$, so that the desired drift limit is satisfied. The fundamental period $T_1$ of the structure to be designed may fall in three different ranges, and result in different design considerations. These are explained on a generic design displacement response spectrum (Fig. 6.1) in the following.

(a) If the allowable displacement limit corresponds to a period limit $T_{max} > T_g$, and the estimated structural fundamental period $T_1$ falls in the range that $T_g < T_1 < T_{max}$, then the required stiffness and strength are satisfied. This case is illustrated in Fig. 6.1, where the displacement limit is given by case 2 and corresponding period limit is designated as $T_{max,2}$, and where the fundamental period of the structure is identified as $T_{1,a}$. For this design situation, the drift limit is satisfied regardless of the provided strength. Therefore, the structural members can be designed and proportioned according to other loading conditions and functional requirements, and detailed to enable them to undergo the deformations resulting from the estimated maximum displacement.

(b) If the allowable displacement limit corresponds to a period limit $T_{max} < T_g$, and the structural period $T_1$ falls in the range such that $T_1 < T_{max} < T_g$ (see displacement limit case 1, $T_{max,1}$, and $T_{1,b}$ in Fig. 6.1), the stiffness will be acceptable only if an appropriate strength coefficient is selected. In some cases it may be more economical to increase the stiffness (decrease $T_1$) because a lower strength will then be required in order to meet the
displacement requirement.

(c) If the period of the structure is found to be longer than the maximum allowable period (e.g., displacement limit case 1, $T_{max,1}$, and $T_{1,c}$ in Fig. 6.1), the structure is considered not sufficiently stiff, and the stiffness must be increased in order to control the maximum displacement response. In this case, it is necessary to modify the preliminary design (make the structure stiffer) and repeat from step (2) until the period of the structure falls into the ranges of case (a) or case (b). This iteration is an extra step in comparison with the strength design approach, but it is an important step in the sense that it connects the displacement response of a structure under the action of a given earthquake directly to the stiffness characteristics of the structure.

(7) Determine the minimum required strength for structures in case (b). This can be done by following in reverse order the procedures to define the generalized yielding strength (App. B). The minimum required strength ratio $\eta^*$ (for predetermined stiffness) is read from the modified design displacement response spectrum such that the displacement limit is satisfied. As defined in App. B,

$$\eta^* = \frac{R_y^*}{M^* A_{g,max}}$$

$$R_y^* = \phi^T \{R_y\} = \phi^T \{P\} R_y$$

in which the deflection shape, $\{\phi\}$, and the load distribution vector, $\{P\}$, are defined in the static analysis. Once the strength ratio $\eta^*$ is determined, the yielding load factor, $R_y$, and the required yielding resistance, $\{R_y\}$ can also be determined. Taking the required yielding resistance as applied lateral loads and combining with other loads, the minimum required strength for each member can be obtained through elastic static analysis. Usually, a structure having strength so determined will have a slightly larger equivalent yielding strength, since the equivalent yielding resistance is subjectively defined based on the overall behavior of the structure rather than the first yielding of any member. If the required member strength is feasible and is satisfied, step (7) of the design method is completed by providing details necessary to achieve the expected deformation demands.
6.4 Examples of Design by Strength and Displacement Methods

Two generic five-story reinforced concrete frames as generally described in Ch. 5 (Fig. 5.1) were used as examples to demonstrate some details of the displacement design approach, and to show the difference of the response that results from the different design methods.

The first example considers a case having a relatively large value of $T_g$, as represented by the S48E component of the 1952 Kern County earthquake recorded at the Santa Barbara Courthouse. The structure is first designed using the UBC [63] strength design method. The reinforced concrete frame is categorized as a special moment-resistant space frame ($R_w = 12$). The structure is sited in seismic zone 4 ($Z = 0.4$) on a very deep soil foundation ($S = 2.0$). (The value $S = 2.0$ is consistent with a site that would experience a ground motion having large value of $T_g$.) A preliminary design for dimensions of members is available as shown in Fig. 6.2. Following the steps as described in Sec. 6.2, the gross-section period of the frame is calculated to be 0.71 sec. Thus, a required minimum base shear strength of approximately 9.2% of the weight is determined. Required flexural strength for each member is then determined through an elastic static analysis, with results as shown in Fig. 6.2. It needs to be noted that the maximum calculated elastic (gross-section stiffness) inter-story drift under the design earthquake loads is 0.22%, which is only 2/3 of the code allowed drift ($0.04/R_w = 0.04/12 = 0.033$).

To determine the likely performance of the structure, it is assumed that structural members have been proportioned so that the provided flexural strengths are the same as those required (Fig. 6.2). Axial compression and shear strengths are considered to be adequate as any failure in these modes should be avoided. In order to account for the cracking of reinforced concrete sections in the analysis, cracked beam sections having half of the gross-section stiffness were used in the calculation of dynamic response under earthquake excitations. The resulting calculated fundamental period was 0.91 sec. The dynamic response calculation was performed using the program ASNR-1 [33], with the Santa Barbara record scaled to 0.4g peak acceleration used as the earthquake input. Other assumptions
as described in Ch. 5 regarding frame analysis were also applied here.

The calculated roof level displacement response history of the frame is plotted in Fig. 6.3 as the solid curve. The deflected shape and the distribution of story drift at the moment of maximum roof displacement are shown in Fig. 6.4. It is observed that a maximum roof displacement of approximately 9 in. and a maximum inter-story drift of 2.4% have resulted. If a story drift index of 1.5% is considered as a desirable limit, a satisfactory performance probably has not been achieved for this frame by the strength design approach. In addition, it is noted that the maximum drift is approximately ten times that calculated under the UBC Code forces.

The elastic responses for the same frame are also calculated and plotted in Fig. 6.3 and 6.4, as reference.

The same frame was redesigned using the displacement design method. An elastic displacement response spectrum for the Santa Barbara record scaled to 0.4g level with 5% critical damping was first calculated and idealized with smooth curves (Fig. 6.5). Assuming the ratio of $L^*/M^*$ to be 1.25 (Table 5.1), a design displacement response spectrum (modified to read roof displacement) was constructed based on the smoothed elastic displacement response spectrum, as shown in Fig. 6.6.

Considering a maximum inter-story drift index of 1.5% as a desirable upper bound and assuming the coefficient of distortion to be 1.5 for a five-story frame (Table 5.6), the maximum roof level displacement of 6 in. is determined. A horizontal line at the displacement of 6 in. is drawn on the modified design displacement response spectrum (Fig. 6.6). Examining the intersections of that horizontal line and the design displacement response spectrum, it is observed that for the given frame having fundamental period of 0.91 sec (assuming cracked beam sections), it would be impossible to control the displacement below the desired drift limit, because of the lack of sufficient stiffness. Therefore, the stiffness of the structure must be increased.

It can be observed from the design displacement response spectrum (Fig. 6.6) that a
minimum stiffness resulting in a fundamental period of 0.75 sec (corresponding to elastic behavior) is required if the drift is to be properly controlled. If the period of the structure is established below this limiting value, then many possible combinations of the stiffness and strength can be selected to make the frame have limited displacement response. One of the possible solutions is to reduce the displacement response solely by making the frame stiffer. Taking the frame designed using the strength design method so far as a preliminary design, the strength ratio, $\eta^*$, was determined to be approximately 0.55. Reading from the design displacement response spectrum, a period of 0.5 sec is required for the given strength ratio of 0.55 so that the displacement of the frame would be controlled below the limit. The dimensions of the beams and columns are then increased (as shown inside parenthesis in Fig. 6.2) and a fundamental period of 0.52 sec (cracked beam sections) is obtained. The strength of members are kept unchanged. The calculated response of the stiffened frame under the same earthquake record shows reduced displacement as desired (shown in dotted lines in Fig. 6.3 and 6.4). The maximum calculated roof displacement is 4.8 in., compared with a maximum target value of 6.0 in.

As another example, a frame having $T > T_s$ is considered. Assume that the structure is founded on a medium firm site (using $S = 1.2$). Using the preliminary design as in the first example and following the code design procedure, a required minimum base shear strength of 6.3% is determined. Members of the frame are then proportioned to provide required flexural strengths. As discussed before, all of the other strength and detailing requirements are assumed to be satisfied.

The same frame is also designed using the displacement design approach. Taking the N21E component of the Taft record of the same 1952 Kern County earthquake (scaled to have 0.4g peak ground acceleration) as a representative design earthquake, an elastic displacement response spectrum with 5% critical damping and the corresponding smoothed spectrum can be determined (Fig. 6.7). As described in Sec. 6.3, a modified design displacement response spectrum is constructed as shown in Fig. 6.8. Reviewing this design displacement response spectrum, it is recognized that the displacement response of the frame
considered ($T_1 = 0.91 \text{ sec with cracked beam sections}$) will be independent of the strength of the structure under the excitation of the earthquake considered ($T_g = 0.7 \text{ sec} < 0.91 \text{ sec}$). Assuming the same displacement limit as in the first example, the minimum stiffness requirement is found to be satisfied (Fig. 6.8). As a consequence, the frame as designed using the strength method can be expected to satisfy the required drift limit. However, according to results presented in Ch. 4 and Ch. 5, the drift is independent of strength, and the same liberty can be applied in the distribution of strength over the structure height. In the present design example, the frame will be modified (from the design using strength approach) by assigning all beams to have uniform strength equal to that of the roof level beams.

Calculated responses of the originally designed (strength design) frame and the modified version (displacement design) under the Taft record are presented in Fig. 6.9 and 6.10. The displacement responses are practically the same for both designs, and both result in maximum inter-story drift less than the prescribed limit of 1.5%. In this example, the calculated roof displacement for the frame designed using the strength method was approximately 2.5 times the value corresponding to the yielding of the structure. Though both frames performed satisfactorily, reduced beam strength in the lower stories based on the displacement design approach resulted in a more cost effective solution to the design problem at hand.

6.5 Summary Discussion

The examples demonstrate that the displacement design approach is practical and effective for multi-story frame structures. The design method emphasizes the important relationship between stiffness, strength, and drift, and provides relatively simple steps to ensure that stiffness and strength are adequately provided. More conventional design methods based on strength requirements do not consistently provide for adequate drift control. In this regard, the displacement design approach is a preferred method of design.

By providing a measure of the drift demand in a structure, the displacement design
method also provides a measure of the required structural and nonstructural details. In the design of a new structure, the techniques described in Ch. 3 can be used to estimate required structural details in beams and columns. In the evaluation of existing structures, the techniques of Ch. 3 can be used with a determination of structural drift as described in Ch. 5 to gage the adequacy of existing structural elements and details. In cases where inadequate details and proportions are identified, the displacement design approach of Ch. 6 provides an effective tool for determining the redesign requirements that will ensure that drift is controlled to within the drift capacities of the existing elements. In many cases of structural redesign, the displacement design approach will identify stiffness as the main deficiency of an existing structure, and will dictate the provision of increased stiffness as opposed to increased strength. Conventional strength design approaches are not likely to identify such cases, and consequently may result in less effective redesigns.
CHAPTER 7

SUMMARY AND CONCLUSIONS

The primary objective of this study has been to develop techniques by which the peak inelastic displacement response of structures subjected to design level earthquakes can be readily estimated, and subsequently, to develop a displacement-based seismic design method. The displacement design method focuses on lateral displacement as being the prime determinant of damage, and aims to realize damage control of structures during strong earthquakes through the control of lateral displacements. Having the lateral displacement of a structure controlled so as to not exceed a pre-established limit, the deformation demand on structural elements can be assessed during the design process so that an efficient design can be achieved.

In the course of the study, the deformation capacities of reinforced concrete beams, columns, and beam-column assemblages were examined by reviewing experimental data. More than one hundred laboratory experiments were reviewed. In order to generalize the observations from the experiments, a parametric study of the plastic hinge rotation capacities of reinforced concrete beams and columns was conducted analytically.

The displacement response characteristics of a range of SDOF systems were studied. A simple method of estimating the peak inelastic displacement response during earthquakes was developed, and the validity of the method was verified by comparison with computed responses of SDOF systems subjected analytically to recorded earthquake ground accelerations.

An equivalent SDOF model representing the displacement response of MDOF systems was developed. The estimate of displacement response of SDOF systems developed previously was extended to MDOF systems using the equivalent SDOF model. The distribution of lateral displacements for reinforced concrete frames having a continuous distribution of stiffness and strength over the height was also studied so that the maximum inter-story
drift in a frame could be assessed based on estimated roof level displacement.

A displacement-based design approach was then outlined in which the estimated displacement or inter-story drift was used as a guideline in selecting proper combinations of the structural stiffness and strength. Design examples were presented to demonstrate the details of the displacement design method.

Based on the results of the current study, the following conclusions are made.

(1) Excluding cases for which shear or anchorage failures occur, and excluding cases for which the axial load ratio \( P/A_f \) is greater than 0.5, reinforced concrete beams and columns designed and constructed according to current practice are capable of undergoing deformations resulting from a lateral inter-story drift on the order of 2% without significant degradation of the load carrying capacity. Beam-column assemblages or frames can be expected to have larger deformation capacities.

(2) Analytical techniques can be used to study the plastic hinge rotation capacities for reinforced concrete members. Trends in the variation of the deformation capacity with respect to different parameters can be observed from such analytical studies. The calculated plastic hinge rotation capacity represents an average of measured values from experiments. Actual rotation capacities under reversed cyclic loadings may be less than calculated values. This possibility should be considered when assessing the detailing requirements for structural members based on calculated results.

(3) The displacement response of SDOF systems subjected to earthquake ground motions can be characterized in two period ranges, divided by the characteristic period of the ground motion, \( T_g \). In the period range of \( T > T_g \), the maximum displacement response of a system is largely independent of the strength of the system, and the peak inelastic displacement responses can be estimated as being equal to the elastic values. In the period range of \( T < T_g \), both stiffness and strength of the system affect the maximum inelastic response. A simple method of estimating the peak inelastic displacements for structures having \( T < T_g \) was developed and shown to be effective in providing an upper bound to the
peak displacement values.

(4) Having the peak inelastic displacement responses defined over the entire period range, a design displacement response spectrum, which is suitable for use in conjunction with conventional elastic analysis, can be readily constructed and incorporated into design practice.

(5) The displacement responses of regular multi-story structures can be effectively modeled by an equivalent SDOF system, and the peak displacement can be estimated using the same techniques developed for SDOF systems. For regularly configured structures, the maximum inter-story drift index during design level earthquakes can be estimated based on estimated maximum roof displacement.

(6) A displacement-based seismic design approach is feasible given a design displacement response spectrum for the design earthquake. The uncertainties involved in establishing the design earthquake and structural modeling are effectively the same for both strength and displacement design methods. The steps required to use the displacement design method are similar to those required to carry out a traditional strength design. Using the displacement design method, the deformation of structures during the design earthquake can be effectively controlled not to exceed the pre-established limit.

(7) The displacement design method provides a direct estimate of the peak inter-story drift index that a structure may experience during the design earthquake. Therefore, the damage to structural and non-structural components, and the detailing requirements of structural members, can be gaged with greater reliability based on a directly assessed displacement or drift limit. The approach is valid for new designs, for evaluating existing designs, and for redesigning existing structures.

(8) The displacement design approach highlights both the stiffness and strength of a structure in resisting earthquake excitations, whereas the stiffness of a structure is not explicitly accounted for in the conventional strength design approach. In particular, the displacement design method demonstrates the greater importance of structural stiffness in a
certain period range. Problems associated with the lack of adequate stiffness can be readily identified and rectified using the displacement design approach. Conventional strength design approaches do not focus attention on structural deformations, and consequently may be less effective in controlling damage during strong earthquakes.
REFERENCES


[2] American Concrete Institute, Committee 318, Building Code Requirements for Reinforced Concrete (ACI 318-89), Detroit, Michigan, 1989

[3] American Concrete Institute, Committee 318, Commentary on Building Code Requirements for Reinforced Concrete, Detroit, Michigan, 1989


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Special Research Paper, Civil Engineering Department, University of Illinois, Urbana, 1985.


Table 2.1 Properties and Deformation Capacities of RC Beams

<table>
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<tr>
<th>Beam ID</th>
<th>Reinforcing Index</th>
<th>Bottom/Top Steel Ratio</th>
<th>Transverse Steel Index</th>
<th>Shear Span Ratio</th>
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University of California at Berkeley, California [23]

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University of California at Berkeley, California [44]

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University of Auckland, New Zealand [15]

| 1A | 0.1691 | 1.00 | 0.0313† | 2.09 | Cyclic-I | 0.0429 |
| 1B | 0.1691 | "    | 0.0231† | 3.01 | "        | 0.0458 |
| 2A | 0.1441 | "    | 0.0180† | 3.93 | "        | 0.0481 |
| 2B | 0.1441 | "    | "†    | 4.85 | "        | 0.0447 |
| 3A | 0.1768 | 0.64 | 0.0231† | 3.01 | "        | 0.0437 |
| 5A | 0.0913 | 1.00 | "†    | 2.99 | "        | 0.0439 |
| 5B | 0.0913 | "    | 0.0313† | 2.08 | "        | 0.0440 |
Table 2.1 Properties and Deformation Capacities of RC Beams (Cont'd)

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<th>Beam ID</th>
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<th>Transverse Steel Index</th>
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1 Cyclic-I = Cyclic loading with increased displacement magnitude;
   Cyclic-C = Cyclic loading with constant displacement magnitude.

2 " sign indicates the same value as the above.

3 Combined cyclic and repeated loading history.

$\dagger$ Beams satisfying the current ACI code [2] detailing requirements.
Table 2.2 Properties and Deformation Capacities of RC Columns

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<tr>
<th>Column ID</th>
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</tr>
<tr>
<td>U3</td>
<td>0.3961</td>
<td>0.141</td>
<td>0.0365†</td>
<td>&quot;</td>
<td>&quot;</td>
<td>0.0510</td>
</tr>
<tr>
<td>U4</td>
<td>0.4388</td>
<td>0.153</td>
<td>0.0672†</td>
<td>&quot;</td>
<td>&quot;</td>
<td>0.0869</td>
</tr>
<tr>
<td>U5</td>
<td>0.2796</td>
<td>-0.08~0.08</td>
<td>0.0130</td>
<td>&quot;</td>
<td>&quot;</td>
<td>0.0442</td>
</tr>
<tr>
<td>U6</td>
<td>0.3765</td>
<td>0.131</td>
<td>0.0452†</td>
<td>&quot;</td>
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<td>0.0895</td>
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<tr>
<td>U7</td>
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<td>0.126</td>
<td>0.0452†</td>
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<td>&quot;</td>
<td>0.0882</td>
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<td>D1</td>
<td>0.3603</td>
<td>0.0</td>
<td>0.0130</td>
<td>&quot;</td>
<td>D</td>
<td>0.0414</td>
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<td>D2</td>
<td>0.4809</td>
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<td>0.0130</td>
<td>&quot;</td>
<td>&quot;</td>
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</tr>
<tr>
<td>D3</td>
<td>0.3961</td>
<td>0.141</td>
<td>0.0365†</td>
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<td>&quot;</td>
<td>0.0288</td>
</tr>
<tr>
<td>D4</td>
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<td>0.112</td>
<td>0.0672†</td>
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<td>D5</td>
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<td>-0.08~0.08</td>
<td>0.0130</td>
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<td>&quot;</td>
<td>0.0247</td>
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<tr>
<td>B1</td>
<td>0.4388</td>
<td>0.153</td>
<td>0.0672†</td>
<td>&quot;</td>
<td>B</td>
<td>0.0563</td>
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<tr>
<td>B2</td>
<td>0.3547</td>
<td>0.124</td>
<td>0.0365†</td>
<td>&quot;</td>
<td>&quot;</td>
<td>0.0533</td>
</tr>
<tr>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>00–U</td>
<td>0.261</td>
<td>0.0</td>
<td>0.0165†</td>
<td>1.73</td>
<td>U</td>
<td>0.0186</td>
</tr>
<tr>
<td>120C–U</td>
<td>0.355</td>
<td>0.187</td>
<td>&quot;†</td>
<td>&quot;</td>
<td>&quot;</td>
<td>0.0167</td>
</tr>
<tr>
<td>120C–B</td>
<td>0.219</td>
<td>0.140</td>
<td>&quot;†</td>
<td>&quot;</td>
<td>B</td>
<td>0.0173</td>
</tr>
<tr>
<td>00–B</td>
<td>0.218</td>
<td>0.0</td>
<td>&quot;†</td>
<td>&quot;</td>
<td>&quot;</td>
<td>0.0174</td>
</tr>
<tr>
<td>50T–U</td>
<td>0.256</td>
<td>-0.068</td>
<td>&quot;†</td>
<td>&quot;</td>
<td>U</td>
<td>0.0226</td>
</tr>
<tr>
<td>100T–U</td>
<td>0.233</td>
<td>-0.124</td>
<td>&quot;†</td>
<td>&quot;</td>
<td>&quot;</td>
<td>0.0225</td>
</tr>
<tr>
<td>200T–U</td>
<td>0.225</td>
<td>-0.239</td>
<td>&quot;†</td>
<td>&quot;</td>
<td>&quot;</td>
<td>0.0233</td>
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<tr>
<td>50T–B</td>
<td>0.340</td>
<td>-0.075</td>
<td>&quot;†</td>
<td>&quot;</td>
<td>B</td>
<td>0.0217</td>
</tr>
<tr>
<td>ATC–U</td>
<td>0.337</td>
<td>-0.15~0.18</td>
<td>&quot;†</td>
<td>&quot;</td>
<td>U</td>
<td>0.0112</td>
</tr>
<tr>
<td>ATC–B</td>
<td>0.316</td>
<td>-0.07~0.17</td>
<td>&quot;†</td>
<td>&quot;</td>
<td>B</td>
<td>0.0173</td>
</tr>
</tbody>
</table>
### Table 2.2 Properties and Deformation Capacities of RC Columns (Cont'd)

<table>
<thead>
<tr>
<th>Column ID</th>
<th>Reinforcing Index $r_i$</th>
<th>Axial Load Index $P/A_g f_c$</th>
<th>Transverse Steel Index $\rho_s \sqrt{b/s}$</th>
<th>Shear Span Ratio $l/d$</th>
<th>Loading Direction</th>
<th>Column End Rotation $\theta_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ONE</td>
<td>0.299</td>
<td>0.26</td>
<td>0.0391†</td>
<td>2.46</td>
<td>U</td>
<td>0.0253</td>
</tr>
<tr>
<td>TWO</td>
<td>0.167</td>
<td>0.214</td>
<td>0.0620†</td>
<td>&quot;</td>
<td>&quot;</td>
<td>0.0213</td>
</tr>
<tr>
<td>THREE</td>
<td>0.322</td>
<td>0.42</td>
<td>0.0504†</td>
<td>&quot;</td>
<td>&quot;</td>
<td>0.0176</td>
</tr>
<tr>
<td>FOUR</td>
<td>0.293</td>
<td>0.76</td>
<td>0.0966†</td>
<td>&quot;</td>
<td>&quot;</td>
<td>0.0129</td>
</tr>
<tr>
<td>University of Canterbury, New Zealand [18]</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>ONE</td>
<td>0.273</td>
<td>0.38</td>
<td>0.0567</td>
<td>4.47</td>
<td>U</td>
<td>0.0253</td>
</tr>
<tr>
<td>Unit 3</td>
<td>0.258</td>
<td>0.21</td>
<td>0.0468</td>
<td>&quot;</td>
<td>&quot;</td>
<td>0.0367</td>
</tr>
<tr>
<td>Unit 4</td>
<td>0.176</td>
<td>0.23</td>
<td>0.0489†</td>
<td>4.47</td>
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<td>0.0253</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Unit 1</td>
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<td>0.43</td>
<td>0.0660†</td>
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<td>&quot;</td>
<td>0.0248</td>
</tr>
<tr>
<td>Unit 2</td>
<td>0.198</td>
<td>0.23</td>
<td>0.0504†</td>
<td>&quot;</td>
<td>&quot;</td>
<td>0.0291</td>
</tr>
<tr>
<td>Unit 3</td>
<td>0.237</td>
<td>0.42</td>
<td>0.0755†</td>
<td>&quot;</td>
<td>&quot;</td>
<td>0.0280</td>
</tr>
<tr>
<td>Unit 4</td>
<td>0.235</td>
<td>0.23</td>
<td>0.0288</td>
<td>&quot;</td>
<td>U</td>
<td>0.0375</td>
</tr>
<tr>
<td>Unit 7</td>
<td>0.166</td>
<td>0.39</td>
<td>0.0415</td>
<td>&quot;</td>
<td>&quot;</td>
<td>0.0233</td>
</tr>
</tbody>
</table>

1. Compression loadings are defined as positive.

2. $U = \text{Cyclic lateral loading along one principal axis of the column;}$

   $D = \text{Cyclic lateral loading along one diagonal of the column;}$

   $B = \text{Cyclic lateral loading along both principal axes of the column.}$

3. The larger value in a principal direction when subjected to biaxial loadings.

4. " sign indicates the same value as the above.

† Columns satisfying the current ACI code [2] detailing requirements.
Table 2.3 Properties and Deformation Capacities of RC Beam-Column Assemblages

<table>
<thead>
<tr>
<th>Assemblage ID</th>
<th>Span to Height Ratios ( L_1/H ) ( L_2/H )</th>
<th>Specimen Type ( \text{Type}^3 )</th>
<th>Loading Direction ( \text{Direction}^4 )</th>
<th>Joint Location</th>
<th>Drift Index ( \delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1</td>
<td>1.02</td>
<td>1</td>
<td>U</td>
<td>Interior</td>
<td>0.0463</td>
</tr>
<tr>
<td>X2</td>
<td>( \text{Type} ) ( \text{Type}^3 )</td>
<td></td>
<td></td>
<td></td>
<td>0.0551</td>
</tr>
<tr>
<td>X3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0568</td>
</tr>
<tr>
<td>S1</td>
<td></td>
<td>3</td>
<td></td>
<td></td>
<td>0.0619</td>
</tr>
<tr>
<td>S2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0616</td>
</tr>
<tr>
<td>S3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0611</td>
</tr>
</tbody>
</table>

The University of Michigan, Ann Arbor, Michigan [13]

| X1            |                                  |                               |                 |                 | 0.0372         |
| X2            |                                  |                               |                 |                 | 0.0425         |
| X3            |                                  |                               |                 |                 | 0.0429         |
| S1            |                                  |                               |                 |                 | 0.0580         |
| S2            |                                  |                               |                 |                 | 0.0520         |
| S3            |                                  |                               |                 |                 | 0.0623         |
| J1            | 1.22                             | 4                             | U               | Interior        | 0.0408         |
| J2            |                                  |                               |                 |                 | 0.0402         |
| J3            |                                  |                               |                 |                 | 0.0406         |

The University of Michigan, Ann Arbor, Michigan [14]

The University of Texas, Austin, Texas [22]
Table 2.3 Properties and Deformation Capacities of RC Beam-Column Assemblages

<table>
<thead>
<tr>
<th>Assemblage ID</th>
<th>Span to Height Ratios</th>
<th>Specimen Type</th>
<th>Loading Direction</th>
<th>Joint Location</th>
<th>Drift Index</th>
</tr>
</thead>
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<tr>
<td></td>
<td>$L_1/H$</td>
<td>$L_2/H$</td>
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<td></td>
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</tr>
<tr>
<td>University of Tokyo, Tokyo, Japan [21]</td>
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<td></td>
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<tr>
<td>S1</td>
<td>1.84</td>
<td>-</td>
<td>2</td>
<td>U</td>
<td>Interior</td>
</tr>
<tr>
<td>S2</td>
<td>&quot;</td>
<td>-</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>J1</td>
<td>&quot;</td>
<td>-</td>
<td>1</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>J2</td>
<td>&quot;</td>
<td>-</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>C1</td>
<td>&quot;</td>
<td>-</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>C2</td>
<td>&quot;</td>
<td>-</td>
<td>&quot;</td>
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<tr>
<td>C3</td>
<td>&quot;</td>
<td>-</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&quot;</td>
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<td>EW2</td>
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<td>2.50</td>
<td>3</td>
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<tr>
<td>EW3</td>
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<td>&quot;</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&quot;</td>
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</tr>
<tr>
<td>J1</td>
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<td>-</td>
<td>1</td>
<td>U</td>
<td>Exterior</td>
</tr>
<tr>
<td>J2</td>
<td>&quot;</td>
<td>0.73</td>
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<td>&quot;</td>
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<tr>
<td>J3</td>
<td>&quot;</td>
<td>-</td>
<td>1</td>
<td>&quot;</td>
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<tr>
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<tr>
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<td>&quot;</td>
<td>&quot;</td>
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<tr>
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<td>0.73</td>
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<tr>
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<td>1.16</td>
<td>3</td>
<td>B</td>
<td>Interior</td>
</tr>
<tr>
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<td>&quot;</td>
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<td>1.06</td>
<td>4</td>
<td>U</td>
<td>Interior</td>
</tr>
</tbody>
</table>

1 $L_1 = $ The beam length in the longitudinal direction,

$L_2 =$ The beam length in the transverse direction.
2 The ratio is \( \frac{2L_1}{H} \) in cases of exterior joint.

3 Type 1 = Beams and columns only in the longitudinal direction.

   Type 2 = Type 1 plus transverse beams.

   Type 3 = Type 2 plus floor slabs.

   Type 4 = Type 1 plus floor slabs.

4 \( U = \) Uniaxial cyclic loading in the longitudinal direction.

   \( B = \) Biaxial cyclic loadings in both longitudinal and transverse directions.

5 The larger value in a principal direction when subjected to biaxial loadings.

6 --- sign means not available or not applicable.

7 " sign indicates the same value as the above.
Table 3.1 Ratios of Calculated and Measured Beam End Rotations

<table>
<thead>
<tr>
<th>Beam ID</th>
<th>Reporter</th>
<th>Measured Rotation $\theta_b$</th>
<th>Calculated Rotation $\theta'_b$</th>
<th>Ratio of Rotations $\theta'_b/\theta_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam R-5</td>
<td>Ma [23]</td>
<td>0.0382</td>
<td>0.0482</td>
<td>1.26</td>
</tr>
<tr>
<td>Beam R-6</td>
<td>¹</td>
<td>0.0459</td>
<td>0.0540</td>
<td>1.18</td>
</tr>
<tr>
<td>Beam T-3</td>
<td>²</td>
<td>0.0503</td>
<td>0.0357</td>
<td>0.71</td>
</tr>
<tr>
<td>Beam 35</td>
<td>Popov [44]</td>
<td>0.0320</td>
<td>0.0251</td>
<td>0.78</td>
</tr>
<tr>
<td>Beam 46</td>
<td>²</td>
<td>0.0320</td>
<td>0.0332</td>
<td>1.04</td>
</tr>
<tr>
<td>Beam 43</td>
<td>²</td>
<td>0.0480</td>
<td>0.0601</td>
<td>1.25</td>
</tr>
<tr>
<td>1A</td>
<td>Fenwick [15]</td>
<td>0.0429</td>
<td>0.0700</td>
<td>1.63</td>
</tr>
<tr>
<td>1B</td>
<td>²</td>
<td>0.0458</td>
<td>0.0553</td>
<td>1.21</td>
</tr>
<tr>
<td>2A</td>
<td>²</td>
<td>0.0481</td>
<td>0.0525</td>
<td>1.09</td>
</tr>
<tr>
<td>2B</td>
<td>²</td>
<td>0.0447</td>
<td>0.0560</td>
<td>1.25</td>
</tr>
<tr>
<td>5A</td>
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<td>0.0439</td>
<td>0.0618</td>
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<td>0.0440</td>
<td>0.0763</td>
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</tr>
<tr>
<td>66-35-RV-5</td>
<td>Brown [6]</td>
<td>0.0720</td>
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<td>0.92</td>
</tr>
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</tr>
<tr>
<td>88-35-RV-5</td>
<td>²</td>
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<tr>
<td>88-32-RV-5</td>
<td>²</td>
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</tr>
</tbody>
</table>

Mean value of $\theta'_b/\theta_b = 1.208$

Standard derivation ($\sigma_{n-1}$) = 0.355

¹ " sign indicates the same report as the above.
Table 3.2 Ratios of Calculated and Measured Column End Rotations

<table>
<thead>
<tr>
<th>Column ID</th>
<th>Reporter [Reference]</th>
<th>Measured Rotation $\theta_c$</th>
<th>Calculated Rotation $\theta'_c$</th>
<th>Ratio of Rotations $\theta'_c/\theta_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>U1</td>
<td>Saatcioglu [48]</td>
<td>0.0461</td>
<td>0.0285</td>
<td>0.62</td>
</tr>
<tr>
<td>U2</td>
<td></td>
<td>0.0422</td>
<td>0.0177</td>
<td>0.42</td>
</tr>
<tr>
<td>U3</td>
<td></td>
<td>0.0510</td>
<td>0.0393</td>
<td>0.77</td>
</tr>
<tr>
<td>U4</td>
<td></td>
<td>0.0869</td>
<td>0.0587</td>
<td>0.67</td>
</tr>
<tr>
<td>U5</td>
<td></td>
<td>0.0442</td>
<td>0.0224</td>
<td>0.51</td>
</tr>
<tr>
<td>U6</td>
<td></td>
<td>0.0895</td>
<td>0.0450</td>
<td>0.50</td>
</tr>
<tr>
<td>U7</td>
<td></td>
<td>0.0882</td>
<td>0.0453</td>
<td>0.51</td>
</tr>
<tr>
<td>Unit 7</td>
<td>Zahn [70]</td>
<td>0.0583</td>
<td>0.0456</td>
<td>0.78</td>
</tr>
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<td>Unit 8</td>
<td></td>
<td>0.0611</td>
<td>0.0414</td>
<td>0.68</td>
</tr>
</tbody>
</table>

Mean value of $\theta'_c/\theta_c = 0.607$

Standard derivation ($\sigma_{n-1}$) = 0.128

1 "" sign indicates the same report as the above.
Table 3.3 Summary of Parameters and Their Values

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Beam Sections</th>
<th>Column Sections</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_t$ (%)</td>
<td>— 1</td>
<td>3.00</td>
</tr>
<tr>
<td>$\rho$ (%)</td>
<td>0.67 ~ 2.50</td>
<td>—</td>
</tr>
<tr>
<td>$\rho'/\rho$</td>
<td>0.25 ~ 1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$\rho_s$ (%)</td>
<td>0.50 ~ 2.00</td>
<td>0.80 ~ 2.80</td>
</tr>
<tr>
<td>$P/A_p f'_c$</td>
<td>—</td>
<td>0.10 ~ 0.60</td>
</tr>
<tr>
<td>$h$ (in)</td>
<td>24</td>
<td>20</td>
</tr>
<tr>
<td>$b/h$</td>
<td>0.5</td>
<td>1.0</td>
</tr>
<tr>
<td>$L/d$</td>
<td>3.0</td>
<td>3.0</td>
</tr>
<tr>
<td>$f'_c$ (ksi)</td>
<td>4.0</td>
<td>4.0</td>
</tr>
<tr>
<td>$f_v$ (ksi)</td>
<td>69.0</td>
<td>69.0</td>
</tr>
</tbody>
</table>

1 — sign indicates not applicable.
Table 4.1 Earthquakes Used for Response Calculation

<table>
<thead>
<tr>
<th>Ground Acceleration Component</th>
<th>Date of Earthquake and Recorded Site</th>
<th>$A_{g,\text{max}}$ (g.)</th>
<th>$T_g$ (sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Castaic N21E</td>
<td>February 9, 1971 San Fernando Earthquake, Castaic Old Ridge Route, CA</td>
<td>0.315</td>
<td>0.35</td>
</tr>
<tr>
<td>Llolleo N10E</td>
<td>March 3, 1985 Chile Earthquake, Llolleo, Chile</td>
<td>0.669</td>
<td>0.50</td>
</tr>
<tr>
<td>Los Angeles N00W</td>
<td>February 9, 1971 San Fernando Earthquake, 8244 Orion Blvd., Los Angeles, CA</td>
<td>0.255</td>
<td>0.65</td>
</tr>
<tr>
<td>Vina Del Mar S20W</td>
<td>March 3, 1985 Chile Earthquake, Vina Del Mar, Chile</td>
<td>0.356</td>
<td>0.70</td>
</tr>
<tr>
<td>Taft N21E</td>
<td>July 21, 1952 Kern County Earthquake, Taft Lincoln School Tunnel, CA</td>
<td>0.156</td>
<td>0.70</td>
</tr>
<tr>
<td>El Centro S00E</td>
<td>May 18, 1940 Imperial Valley Earthquake, El Centro Site Imperial Valley Irrigation District, CA</td>
<td>0.348</td>
<td>0.85</td>
</tr>
<tr>
<td>Miyagi NS</td>
<td>June 2, 1978 Miyagi-Ken-Oki Earthquake, Tohoku University, Sendai, Japan</td>
<td>0.260</td>
<td>0.95</td>
</tr>
<tr>
<td>Santa Barbara S48E</td>
<td>July 21, 1952 Kern County Earthquake, Santa Barbara Courthouse, CA</td>
<td>0.131</td>
<td>1.05</td>
</tr>
<tr>
<td>Derived Pacoima Dam</td>
<td>February 9, 1971 San Fernando Earthquake, Pacoima Dam, CA</td>
<td>0.40</td>
<td>1.50</td>
</tr>
</tbody>
</table>
Table 5.1  Summary of Generalized Quantities for Five-Story Frames

<table>
<thead>
<tr>
<th></th>
<th>Type I $R^1 = 1.2$</th>
<th>Type II $R = 1.2$</th>
<th>Type III $R = 1.2$</th>
<th>Type Ia $R = 1.2$</th>
<th>Type IIa $R = 1.2$</th>
<th>Type IIIa $R = 1.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deflection</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Shape</td>
<td>0.892</td>
<td>0.898</td>
<td>0.960</td>
<td>0.889</td>
<td>0.913</td>
<td>0.957</td>
</tr>
<tr>
<td>at 1% of $AD^2$</td>
<td>0.706</td>
<td>0.707</td>
<td>0.827</td>
<td>0.696</td>
<td>0.726</td>
<td>0.840</td>
</tr>
<tr>
<td>${\phi}$</td>
<td>0.214</td>
<td>0.210</td>
<td>0.266</td>
<td>0.211</td>
<td>0.196</td>
<td>0.289</td>
</tr>
<tr>
<td>$D_y$</td>
<td>1.24</td>
<td>0.70</td>
<td>0.56</td>
<td>2.17</td>
<td>1.28</td>
<td>0.99</td>
</tr>
<tr>
<td>$R_y$</td>
<td>9.85</td>
<td>5.57</td>
<td>4.52</td>
<td>17.19</td>
<td>10.13</td>
<td>7.88</td>
</tr>
<tr>
<td>$M^*$</td>
<td>0.795</td>
<td>0.798</td>
<td>0.934</td>
<td>0.786</td>
<td>0.813</td>
<td>0.953</td>
</tr>
<tr>
<td>$L^*$</td>
<td>1.019</td>
<td>1.020</td>
<td>1.127</td>
<td>1.011</td>
<td>1.026</td>
<td>1.145</td>
</tr>
<tr>
<td>$L^<em>/M^</em>$</td>
<td>1.28</td>
<td>1.28</td>
<td>1.21</td>
<td>1.29</td>
<td>1.26</td>
<td>1.20</td>
</tr>
<tr>
<td>$R_y^*$</td>
<td>116.2</td>
<td>66.0</td>
<td>57.6</td>
<td>202.4</td>
<td>121.2</td>
<td>101.2</td>
</tr>
<tr>
<td>$\omega^*$</td>
<td>10.86</td>
<td>10.55</td>
<td>10.49</td>
<td>10.90</td>
<td>10.79</td>
<td>10.35</td>
</tr>
<tr>
<td>$T^*$</td>
<td>0.58</td>
<td>0.59</td>
<td>0.60</td>
<td>0.58</td>
<td>0.58</td>
<td>0.61</td>
</tr>
</tbody>
</table>

1 $R = \Sigma M_{col}/\Sigma M_{beam}$

2 Average Drift Index, $ADI = \Delta_{roof}/H \times 100\%$
Table 5.2 Summary of Generalized Quantities for Ten-Story Frames

<table>
<thead>
<tr>
<th></th>
<th>Type I $R^1 = 0.8$</th>
<th>Type I $R = 1.2$</th>
<th>Type I $R = 2.0$</th>
<th>Type II $R = 1.2$</th>
<th>Type III $R = 1.2$</th>
<th>Type III $R = 2.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deflection</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Shape at 1% of $ADI^2$</td>
<td>0.985</td>
<td>0.985</td>
<td>0.978</td>
<td>0.990</td>
<td>0.993</td>
<td>0.991</td>
</tr>
<tr>
<td></td>
<td>0.985</td>
<td>0.985</td>
<td>0.978</td>
<td>0.990</td>
<td>0.993</td>
<td>0.991</td>
</tr>
<tr>
<td></td>
<td>0.985</td>
<td>0.985</td>
<td>0.978</td>
<td>0.990</td>
<td>0.993</td>
<td>0.991</td>
</tr>
<tr>
<td></td>
<td>0.985</td>
<td>0.985</td>
<td>0.978</td>
<td>0.990</td>
<td>0.993</td>
<td>0.991</td>
</tr>
<tr>
<td>${\phi}$</td>
<td>0.905</td>
<td>0.895</td>
<td>0.860</td>
<td>0.926</td>
<td>0.958</td>
<td>0.941</td>
</tr>
<tr>
<td></td>
<td>0.825</td>
<td>0.807</td>
<td>0.759</td>
<td>0.843</td>
<td>0.910</td>
<td>0.876</td>
</tr>
<tr>
<td></td>
<td>0.721</td>
<td>0.694</td>
<td>0.637</td>
<td>0.727</td>
<td>0.824</td>
<td>0.775</td>
</tr>
<tr>
<td></td>
<td>0.595</td>
<td>0.561</td>
<td>0.500</td>
<td>0.585</td>
<td>0.694</td>
<td>0.638</td>
</tr>
<tr>
<td></td>
<td>0.453</td>
<td>0.417</td>
<td>0.356</td>
<td>0.428</td>
<td>0.524</td>
<td>0.475</td>
</tr>
<tr>
<td></td>
<td>0.292</td>
<td>0.265</td>
<td>0.212</td>
<td>0.266</td>
<td>0.329</td>
<td>0.302</td>
</tr>
<tr>
<td></td>
<td>0.126</td>
<td>0.115</td>
<td>0.083</td>
<td>0.114</td>
<td>0.135</td>
<td>0.132</td>
</tr>
<tr>
<td>$D_y$</td>
<td>2.40</td>
<td>2.64</td>
<td>2.81</td>
<td>1.36</td>
<td>1.16</td>
<td>1.22</td>
</tr>
<tr>
<td>$R_y$</td>
<td>2.30</td>
<td>2.53</td>
<td>2.67</td>
<td>1.31</td>
<td>1.12</td>
<td>1.19</td>
</tr>
<tr>
<td>$M^*$</td>
<td>1.730</td>
<td>1.672</td>
<td>1.546</td>
<td>1.750</td>
<td>1.945</td>
<td>1.842</td>
</tr>
<tr>
<td>$L^*$</td>
<td>2.131</td>
<td>2.079</td>
<td>1.963</td>
<td>2.129</td>
<td>2.284</td>
<td>2.208</td>
</tr>
<tr>
<td>$L^<em>/M^</em>$</td>
<td>1.231</td>
<td>1.243</td>
<td>1.269</td>
<td>1.217</td>
<td>1.174</td>
<td>1.198</td>
</tr>
<tr>
<td>$R_y^*$</td>
<td>105.5</td>
<td>114.4</td>
<td>116.4</td>
<td>60.4</td>
<td>54.4</td>
<td>56.1</td>
</tr>
<tr>
<td>$\omega^*$</td>
<td>5.04</td>
<td>5.11</td>
<td>5.18</td>
<td>5.04</td>
<td>4.89</td>
<td>5.00</td>
</tr>
<tr>
<td>$T^*$</td>
<td>1.25</td>
<td>1.23</td>
<td>1.21</td>
<td>1.25</td>
<td>1.28</td>
<td>1.26</td>
</tr>
</tbody>
</table>

1 $R = \Sigma M_{col}/\Sigma M_{beam}$

2 Average Drift Index, $ADI = \Delta_{roof}/H \times 100\%$
Table 5.3 Displacement Response of 5-Story Frames having $T_1 > T_g$

<table>
<thead>
<tr>
<th>Earthquakes and Types of Resistance</th>
<th>5-STORY ($T_1 = 0.57$ sec)</th>
<th>$\Delta_{roo}^1$</th>
<th>$\eta^*$</th>
<th>$\Delta_{sdof}^2$</th>
<th>$\Delta_{est}^3$</th>
<th>$\Delta_{roo}^1/\Delta_{sdof}$</th>
<th>$\Delta_{roo}^1/\Delta_{est}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Castaic, Type I, $R^4 = 1.2$</td>
<td></td>
<td>6.34</td>
<td>0.23</td>
<td>7.63</td>
<td>7.63</td>
<td>0.831</td>
<td></td>
</tr>
<tr>
<td>Type II, $R = 1.2$</td>
<td></td>
<td>6.34</td>
<td>0.13</td>
<td>7.63</td>
<td>7.63</td>
<td>0.831</td>
<td></td>
</tr>
<tr>
<td>Type III, $R = 1.2$</td>
<td></td>
<td>6.82</td>
<td>0.10</td>
<td>7.21</td>
<td>7.21</td>
<td>0.946</td>
<td></td>
</tr>
</tbody>
</table>

$^1 \Delta_{roo} = \text{peak roof displacement from frame analysis}$

$^2 \Delta_{sdof} = \text{peak displacement from equivalent SDOF analysis}$

$^3 \Delta_{est} = \text{estimated roof displacement according to elastic response}$

$^4 R = \sum M_{col}/\sum M_{beam}$

$^5 -$ sign indicates the response was not calculated.

$\eta^* = \text{strength ratio of the equivalent SDOF system}$
Table 5.4 Displacement Response of 10-Story Frames having $T_1 > T_g$

<table>
<thead>
<tr>
<th>Earthquakes and Types of Resistance</th>
<th>10-STORY ($T_1 = 1.19$ sec)</th>
<th>$\Delta_{\text{roof}}^1$</th>
<th>$\eta^*$</th>
<th>$\Delta_{\text{sdof}}^2$</th>
<th>$\Delta_{\text{est}}^3$</th>
<th>$\Delta_{\text{roof}}/\Delta_{\text{sdof}}$</th>
<th>$\Delta_{\text{roof}}/\Delta_{\text{est}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Castaic, Type I, $R^4 = 1.2$</td>
<td></td>
<td>12.0</td>
<td>0.087</td>
<td>-5</td>
<td>14.9</td>
<td>-</td>
<td>0.805</td>
</tr>
<tr>
<td>Type II, $R = 1.2$</td>
<td></td>
<td>12.4</td>
<td>0.044</td>
<td>-</td>
<td>14.6</td>
<td>-</td>
<td>0.849</td>
</tr>
<tr>
<td>Type III, $R = 1.2$</td>
<td></td>
<td>12.9</td>
<td>0.035</td>
<td>-</td>
<td>14.1</td>
<td>-</td>
<td>0.915</td>
</tr>
<tr>
<td>El Centro, Type I, $R = 1.2$</td>
<td></td>
<td>10.1</td>
<td>0.196</td>
<td>-</td>
<td>14.9</td>
<td>-</td>
<td>0.678</td>
</tr>
<tr>
<td>Type II, $R = 1.2$</td>
<td></td>
<td>10.5</td>
<td>0.099</td>
<td>-</td>
<td>14.6</td>
<td>-</td>
<td>0.719</td>
</tr>
<tr>
<td>Type III, $R = 1.2$</td>
<td></td>
<td>9.62</td>
<td>0.080</td>
<td>-</td>
<td>14.1</td>
<td>-</td>
<td>0.682</td>
</tr>
<tr>
<td>Los Angeles, Type I, $R = 1.2$</td>
<td></td>
<td>10.2</td>
<td>0.303</td>
<td>-</td>
<td>14.9</td>
<td>-</td>
<td>0.685</td>
</tr>
<tr>
<td>Type II, $R = 2.0$</td>
<td></td>
<td>10.2</td>
<td>0.333</td>
<td>-</td>
<td>15.2</td>
<td>-</td>
<td>0.671</td>
</tr>
<tr>
<td>Type III, $R = 1.2$</td>
<td></td>
<td>13.4</td>
<td>0.153</td>
<td>-</td>
<td>14.6</td>
<td>-</td>
<td>0.918</td>
</tr>
<tr>
<td>Type III, $R = 1.2$</td>
<td></td>
<td>13.0</td>
<td>0.123</td>
<td>-</td>
<td>14.1</td>
<td>-</td>
<td>0.922</td>
</tr>
<tr>
<td>Type III, $R = 2.0$</td>
<td></td>
<td>13.0</td>
<td>0.135</td>
<td>-</td>
<td>14.4</td>
<td>-</td>
<td>0.903</td>
</tr>
<tr>
<td>Miyagi, Type I, $R = 0.8$</td>
<td></td>
<td>8.86</td>
<td>0.433</td>
<td>-</td>
<td>14.8</td>
<td>-</td>
<td>0.599</td>
</tr>
<tr>
<td>Type I, $R = 1.2$</td>
<td></td>
<td>8.57</td>
<td>0.486</td>
<td>9.99</td>
<td>14.9</td>
<td>0.858</td>
<td>0.575</td>
</tr>
<tr>
<td>Type I, $R = 2.0$</td>
<td></td>
<td>8.62</td>
<td>0.535</td>
<td>-</td>
<td>15.2</td>
<td>-</td>
<td>0.567</td>
</tr>
<tr>
<td>Type II, $R = 1.2$</td>
<td></td>
<td>6.44</td>
<td>0.245</td>
<td>7.51</td>
<td>14.6</td>
<td>0.858</td>
<td>0.441</td>
</tr>
<tr>
<td>Type III, $R = 1.2$</td>
<td></td>
<td>7.15</td>
<td>0.197</td>
<td>8.52</td>
<td>14.1</td>
<td>0.839</td>
<td>0.507</td>
</tr>
<tr>
<td>Type III, $R = 2.0$</td>
<td></td>
<td>6.85</td>
<td>0.216</td>
<td>-</td>
<td>14.4</td>
<td>-</td>
<td>0.476</td>
</tr>
</tbody>
</table>

1 $\Delta_{\text{roof}}$ = peak roof displacement from frame analysis

2 $\Delta_{\text{sdof}}$ = peak displacement from equivalent SDOF analysis

3 $\Delta_{\text{est}}$ = estimated roof displacement according to elastic response

4 $R = \sum M_{\text{col}} / \sum M_{\text{beam}}$

5 — sign indicates the response was not calculated.

$\eta^*$ = strength ratio of the equivalent SDOF system
Table 5.5 Displacement Response of 5-Story Frames having $T_1 < T_g$

<table>
<thead>
<tr>
<th>Earthquakes and Types of Resistance</th>
<th>5-STORY ($T_1 = 0.57$ sec)</th>
<th>$\Delta^{1}_{roof}$</th>
<th>$\eta^*$</th>
<th>$\Delta^{2}_{sdof}$</th>
<th>$\Delta^{3}_{est}$</th>
<th>$\Delta_{roof}/\Delta_{sdof}$</th>
<th>$\Delta_{roof}/\Delta_{est}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>El Centro, Type I, $R = 1.2$</td>
<td></td>
<td>4.09</td>
<td>0.51</td>
<td>5</td>
<td>6.49</td>
<td>—</td>
<td>0.630</td>
</tr>
<tr>
<td>Type II, $R = 1.2$</td>
<td></td>
<td>4.31</td>
<td>0.29</td>
<td>—</td>
<td>7.47</td>
<td>—</td>
<td>0.577</td>
</tr>
<tr>
<td>Type III, $R = 1.2$</td>
<td></td>
<td>4.96</td>
<td>0.21</td>
<td>—</td>
<td>7.83</td>
<td>—</td>
<td>0.633</td>
</tr>
<tr>
<td>Miyagi, Type I, $R^4 = 1.2$</td>
<td></td>
<td>10.0</td>
<td>0.48</td>
<td>10.6</td>
<td>22.8</td>
<td>0.945</td>
<td>0.439</td>
</tr>
<tr>
<td>Type II, $R = 1.2$</td>
<td></td>
<td>14.7</td>
<td>0.27</td>
<td>15.2</td>
<td>27.1</td>
<td>0.967</td>
<td>0.542</td>
</tr>
<tr>
<td>Type III, $R = 1.2$</td>
<td></td>
<td>14.6</td>
<td>0.20</td>
<td>16.0</td>
<td>27.1</td>
<td>0.913</td>
<td>0.539</td>
</tr>
<tr>
<td>Type Ia, $R = 1.2$</td>
<td></td>
<td>7.86</td>
<td>0.84</td>
<td>—</td>
<td>17.0</td>
<td>—</td>
<td>0.462</td>
</tr>
<tr>
<td>Type IIa, $R = 1.2$</td>
<td></td>
<td>9.52</td>
<td>0.49</td>
<td>—</td>
<td>22.3</td>
<td>—</td>
<td>0.427</td>
</tr>
<tr>
<td>Type IIIa, $R = 1.2$</td>
<td></td>
<td>11.2</td>
<td>0.35</td>
<td>—</td>
<td>23.8</td>
<td>—</td>
<td>0.471</td>
</tr>
<tr>
<td>Santa, Type I, $R = 1.2$</td>
<td></td>
<td>17.5</td>
<td>0.38</td>
<td>18.0</td>
<td>20.8</td>
<td>0.972</td>
<td>0.842</td>
</tr>
<tr>
<td>Barbara, Type II, $R = 1.2$</td>
<td></td>
<td>18.0</td>
<td>0.21</td>
<td>18.9</td>
<td>24.4</td>
<td>0.952</td>
<td>0.738</td>
</tr>
<tr>
<td>Type III, $R = 1.2$</td>
<td></td>
<td>17.7</td>
<td>0.16</td>
<td>20.4</td>
<td>24.3</td>
<td>0.868</td>
<td>0.728</td>
</tr>
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</table>

1. $\Delta_{roof}$ = peak roof displacement from frame analysis
2. $\Delta_{sdof}$ = peak displacement from equivalent SDOF analysis
3. $\Delta_{est}$ = estimated roof displacement according to Eq. (4.10)
4. $R = \sum M_{col}/\sum M_{beam}$
5. — sign indicates the response was not calculated.

$\eta^*$ = strength ratio of the equivalent SDOF system
Table 5.6 Maximum Inter–Story Drift of Five–Story Frames

<table>
<thead>
<tr>
<th>Earthquakes</th>
<th>Type of Resistance</th>
<th>Dynamic Resistance Analysis</th>
<th>Dynamic Resistance Analysis</th>
<th>Static Resistance Analysis</th>
<th>$\frac{COD_{\text{Static}}}{COD_{\text{Static, Analysis}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Castaic, R = 1.2</td>
<td>Type I</td>
<td>1.06</td>
<td>1.48</td>
<td>1.20</td>
<td>1.23</td>
</tr>
<tr>
<td></td>
<td>Type II</td>
<td>1.06</td>
<td>1.43</td>
<td>1.27</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
<td>Type III</td>
<td>1.14</td>
<td>1.75</td>
<td>1.55</td>
<td>1.13</td>
</tr>
<tr>
<td>El Centro, R = 1.2</td>
<td>Type I</td>
<td>0.68</td>
<td>1.55</td>
<td>1.20</td>
<td>1.29</td>
</tr>
<tr>
<td></td>
<td>Type II</td>
<td>0.72</td>
<td>1.43</td>
<td>1.27</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
<td>Type III</td>
<td>0.83</td>
<td>1.72</td>
<td>1.55</td>
<td>1.11</td>
</tr>
<tr>
<td>Miyagi, R = 1.2</td>
<td>Type I</td>
<td>1.67</td>
<td>1.53</td>
<td>1.20</td>
<td>1.28</td>
</tr>
<tr>
<td></td>
<td>Type II</td>
<td>2.45</td>
<td>1.31</td>
<td>1.27</td>
<td>1.03</td>
</tr>
<tr>
<td></td>
<td>Type III</td>
<td>2.43</td>
<td>1.63</td>
<td>1.55</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>Type Ia</td>
<td>1.31</td>
<td>1.37</td>
<td>1.24</td>
<td>1.10</td>
</tr>
<tr>
<td></td>
<td>Type IIa</td>
<td>1.59</td>
<td>1.44</td>
<td>1.36</td>
<td>1.07</td>
</tr>
<tr>
<td></td>
<td>Type IIIa</td>
<td>1.86</td>
<td>1.68</td>
<td>1.56</td>
<td>1.08</td>
</tr>
<tr>
<td>Santa, R = 1.2</td>
<td>Type I</td>
<td>2.92</td>
<td>1.45</td>
<td>1.20</td>
<td>1.21</td>
</tr>
<tr>
<td>Barbara, R = 1.2</td>
<td>Type II</td>
<td>3.00</td>
<td>1.29</td>
<td>1.27</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td>Type III</td>
<td>2.95</td>
<td>1.56</td>
<td>1.55</td>
<td>1.01</td>
</tr>
</tbody>
</table>

1 Average Drift Index, $ADI = \frac{\Delta_{\text{roof}}}{H} \times 100\%$

2 Story Drift Index, $SDI = \frac{(\Delta_i - \Delta_{i-1})}{h_i} \times 100\%$

3 Coefficient of Distortion, $COD = \frac{SDI}{ADI}_{\text{max}}$

4 $R = \frac{\sum M_{\text{col}}}{\sum M_{\text{beam}}}$
Table 5.7 Maximum Inter-Story Drift of Ten-Story Frames

<table>
<thead>
<tr>
<th>Earthquakes</th>
<th>Type of Resistance</th>
<th>$ADI^{1,2}$</th>
<th>$COD^3$</th>
<th>$COD$</th>
<th>$COD_{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dynamic Analysis</td>
<td>Dynamic Analysis</td>
<td>Static Analysis</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Castaic,</td>
<td>Type I, $R = 1.2$</td>
<td>1.00</td>
<td>1.92</td>
<td>1.52</td>
<td>1.26</td>
</tr>
<tr>
<td></td>
<td>Type II, $R = 1.2$</td>
<td>1.09</td>
<td>1.99</td>
<td>1.62</td>
<td>1.23</td>
</tr>
<tr>
<td></td>
<td>Type III, $R = 1.2$</td>
<td>1.08</td>
<td>2.52</td>
<td>1.95</td>
<td>1.29</td>
</tr>
<tr>
<td>El Centro,</td>
<td>Type I, $R = 1.2$</td>
<td>0.84</td>
<td>1.77</td>
<td>1.52</td>
<td>1.16</td>
</tr>
<tr>
<td></td>
<td>Type II, $R = 1.2$</td>
<td>0.87</td>
<td>2.14</td>
<td>1.62</td>
<td>1.32</td>
</tr>
<tr>
<td></td>
<td>Type III, $R = 1.2$</td>
<td>0.80</td>
<td>2.77</td>
<td>1.95</td>
<td>1.42</td>
</tr>
<tr>
<td>Miyagi,</td>
<td>Type I, $R = 0.8$</td>
<td>0.74</td>
<td>2.11</td>
<td>1.66</td>
<td>1.27</td>
</tr>
<tr>
<td></td>
<td>Type I, $R = 1.2$</td>
<td>0.71</td>
<td>1.93</td>
<td>1.52</td>
<td>1.27</td>
</tr>
<tr>
<td></td>
<td>Type I, $R = 2.0$</td>
<td>0.72</td>
<td>1.77</td>
<td>1.44</td>
<td>1.23</td>
</tr>
<tr>
<td></td>
<td>Type II, $R = 1.2$</td>
<td>0.57</td>
<td>1.95</td>
<td>1.62</td>
<td>1.20</td>
</tr>
<tr>
<td></td>
<td>Type III, $R = 1.2$</td>
<td>0.60</td>
<td>2.43</td>
<td>1.95</td>
<td>1.25</td>
</tr>
<tr>
<td></td>
<td>Type III, $R = 2.0$</td>
<td>0.57</td>
<td>2.08</td>
<td>1.73</td>
<td>1.20</td>
</tr>
<tr>
<td>Los Angeles,</td>
<td>Type I, $R = 1.2$</td>
<td>0.85</td>
<td>1.96</td>
<td>1.52</td>
<td>1.29</td>
</tr>
<tr>
<td></td>
<td>Type I, $R = 2.0$</td>
<td>0.85</td>
<td>1.62</td>
<td>1.44</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
<td>Type II, $R = 1.2$</td>
<td>1.11</td>
<td>1.73</td>
<td>1.62</td>
<td>1.07</td>
</tr>
<tr>
<td></td>
<td>Type III, $R = 1.2$</td>
<td>1.08</td>
<td>2.35</td>
<td>1.95</td>
<td>1.21</td>
</tr>
<tr>
<td></td>
<td>Type III, $R = 2.0$</td>
<td>1.08</td>
<td>1.96</td>
<td>1.73</td>
<td>1.13</td>
</tr>
</tbody>
</table>

1 Average Drift Index, $ADI = \Delta_{roof}/H \times 100\%$

2 Story Drift Index, $SDI = (\Delta_i - \Delta_{i-1})/h_i \times 100\%$

3 Coefficient of Distortion, $COD = (SDI/ADI)_{max}$

4 $R = \sum M_{col}/\sum M_{beam}$
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(a) Fixed-Fixed Column Test

(b) Symmetric Column Test
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Fig. 3.2 Stress–Strain Relations for Reinforcing Steel and Concrete
Fig. 3.3 Maximum Measured Concrete Strain versus Confinement Stress
(a) Column segment

\( \begin{align*}
P & \quad \downarrow \\
M & \quad \uparrow \\
A_s & \quad A_s'
\end{align*} \)

(b) Strain distribution

\( \begin{align*}
\varepsilon_{c,max} & \quad \phi_u \\
\varepsilon_{ss} & \quad \varepsilon_{sd} \\
& \quad c \\
& \quad d
\end{align*} \)

(c) Force Distribution

\( \begin{align*}
T_s & \quad \downarrow \\
M & \quad \uparrow \\
Cc & \quad Cs
\end{align*} \)

(d) Summary

\[
\begin{align*}
A_e &= A_s' \\
\varepsilon_s &= \varepsilon_y \\
T_s &\approx C_s \\
c &= F(\varepsilon_{c,max}, P, d) \\
\phi_u &= G(\varepsilon_{c,max}, P, d)
\end{align*}
\]

Fig. 3.4 Axial Load Equilibrium in a Column Section
(a) Typical beam section

(b) Typical column section

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Fig. 3.6 Calculated Plastic Hinge Rotations for Beams, (a) $\rho = 0.67\%$
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Typical Rect. Beams, $\rho = 1.33\%$

Fig. 3.6 Calculated Plastic Hinge Rotations for Beams, (c) $\rho = 1.33\%$
Fig. 3.6 Calculated Plastic Hinge Rotations for Beams, \( \rho = 1.67\% \)
Fig. 3.6 Calculated Plastic Hinge Rotations for Beams, (e) $\rho = 2.00\%$
Typical Rect. Beams, $\rho = 2.50\%$

Fig. 3.6 Calculated Plastic Hinge Rotations for Beams, (f) $\rho = 2.50\%$
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Fig. 4.4 Characteristic Period of 1985 N10E Lolleo Record
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Fig. 4.9 Characteristic Period of 1978 NS Miyagi Record
Fig. 4.10 Characteristic Period of 1952 S48E Santa Barbara Record
Fig. 4.11 Characteristic Period of Derived Pacoima Dam Record
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Fig. 4.13 Estimated and Calculated Responses to 1985 N10E Lolleo Record
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Fig. 4.15 Estimated and Calculated Responses to 1985 S20E Vina del Mar Record
Fig. 4.16 Estimated and Calculated Responses to 1952 N21E Taft Record
Fig. 4.17 Estimated and Calculated Responses to 1940 S00E El Centro Record
Fig. 4.18 Estimated and Calculated Responses to 1978 NS Miyagi Record
Fig. 4.19 Estimated and Calculated Responses to 1952 S48E Santa Barbara Record
Fig. 4.20  Estimated and Calculated Responses to Derived Pacoima Dam Record
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Average Drift Index = 1.0 %

Fig. 5.5 Deflection Shapes for Five-Story Frames
Average Drift Index = 1.0 %

Fig. 5.6 Deflection Shapes for Ten-Story Frames
Fig. 5.7 Calculated Displacement Responses to 1971 N21E Castaic Record, Five-Story Frames
Fig. 5.8 Calculated Displacement Responses to 1978 NS Miyagi Record, Ten-Story Frames
Fig. 5.9 Calculated Displacement Responses to 1978 NS Miyagi Record, Five-Story Frames
Fig. 5.10 Normalized Deflection Shapes at Different Magnitude of $ADI$, Five-Story Frames
Fig. 5.11 Normalized Deflection Shapes at Different Magnitude of ADI, Ten-Story Frames
Fig. 5.12 Deflection Shape and Drift Distribution at Maximum $SDI$, Five-Story Frames Subjected to 1971 N21S Castaic Record
Fig. 5.13 Deflection Shape and Drift Distribution at Maximum SDI, Ten-Story Frames Subjected to 1978 NS Miyagi Record
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Fig. 6.1 A Generic Design Displacement Response Spectrum
Santa Barbara S48E Record, $A_g = 0.4g$, $\xi = 0.05$

Fig. 6.3 Calculated Roof Displacement Responses, Frame Example #1
\[ T_{\text{gross section}} = 0.71 \text{ sec (0.41 sec)} \]
\[ T_{\text{cracked beam}} = 0.91 \text{ sec (0.52 sec)} \]

Fig. 6.2 Dimensions and Flexural Strength of Sample Frames
Fig. 6.4 Normalized Deflection Shapes and Distribution of Inter-Story Drift, Frame Example #1
Fig. 6.5 Calculated and Idealized Elastic Displacement Response Spectra, 1952 S48W Santa Barbara Record

\[ A_{g, \text{max}} = 0.4 \text{ g}, \quad \xi = 0.05, \quad T_g = 1.05 \text{ sec} \]
Fig. 6.6 Modified Design Displacement Response Spectrum, 1952 S48W Santa Barbara Record
Fig. 6.7 Calculated and Idealized Elastic Displacement Response Spectra, 1952 N21E Taft Record

\[ A_{g,\text{max}} = 0.4 \text{ g}, \quad \xi = 0.05, \quad T_g = 0.70 \text{ sec} \]
Fig. 6.8 Modified Design Displacement Response Spectrum, 1952 N21E Taft Record

- $A_{g,\text{max}} = 0.4$ g, $\xi = 0.05$, $L^*/M^* = 1.25$
- Average drift index $= 1.0$ %
Fig. 6.9 Calculated Roof Displacement Responses, Frame Example #2

Taft N21E Record, $A_g = 0.4g$, $\xi = 0.05$
Fig. 6.10 Normalized Deflection Shape and Distribution of Inter-Story Drift, Frame Example #2
Appendix A

Details of the Survey of California Engineers

A survey of engineers' expectations of the seismic performance of reinforced concrete buildings, designed and constructed with the modern technology of earthquake-resistant design, was conducted in June, 1989. A group of fifty engineers in the San Francisco Bay Area and in the Los Angeles Area in California was surveyed. Because the responses of well engineered buildings are of interest, the engineers who were selected in the survey are believed to be knowledgeable in the state of the art of earthquake-resistant design, and to be well experienced in the area of seismic design of building structures. It is believed that such a group of engineers is best suited to provide the answer. The list of engineers selected was by no means exhaustive.

The questions posed in the survey were general. Buildings considered were limited to conventional medium-height reinforced concrete office buildings. Earthquakes were defined as having different return period, and the definitions were briefly explained in the covering letter (Fig. A.1). It was also emphasized in the letter that only the opinions from each individual engineer were of interest.

The details of the surveying questionnaire are shown in Fig. A.2. Totally, thirty-five (out of fifty) engineers responded. Except for one late response received in August, all responses were received within a month.
March 31, 1989

Dear Joe:

As part of a graduate research project to study new design techniques to achieve acceptable performance of reinforced concrete buildings during earthquakes, one of my graduate students and I began to debate just what the structural engineering profession views as acceptable performance for various earthquakes. The answer to this question is critical to the successful completion of research. Because the feel that the profession is best suited to provide the answer, we are conducting a survey of several West Coast engineers. We hope you will take a few minutes now to complete and return the enclosed survey card.

In completing the survey card, please note the following:

1) We are interested in your personal opinion as to the minimum expected performance of a conventional medium-height reinforced concrete office building that you design. We do not want you to recite previously published criteria, but rather to provide your own opinion.

2) An earthquake with recurrence interval at ten years is one that is likely to occur several times during the life of the building. An earthquake with recurrence interval of 500 years is not likely to occur during the life of the building, but possibly might. The range in between is to your own judgment.

We thank you in advance for your response.

Sincerely,

Jack P. Moehle
Associate Professor of
Civil Engineering

Enclosure

Fig. A.1 Covering Letter Attached to the Survey Questionnaire
Place a check below to indicate the minimum expected performance of a building for EQs with different recurrence intervals.

<table>
<thead>
<tr>
<th>Expected recurrence interval</th>
<th>No damage</th>
<th>Light damage minor repair</th>
<th>Serious damage major repair</th>
<th>Not repairable</th>
<th>Total collapse</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 years</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50 years</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100 years</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>500 years</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Comments: ___________________

Your Name and Affiliation: ____________________________________________

(Optional)

Fig. A.2 Format of the Survey Questionnaire
Appendix B

An Equivalent SDOF Model for MDOF Systems

The simplicity of the analysis (both linear and non-linear) of single-degree-of-freedom (SDOF) systems and the availability of well established methods to conduct the analysis have made SDOF systems very useful tools for understanding the characteristics of dynamic response of structural systems. Although not completely accurate, the study of SDOF systems that closely represent the overall behavior of multi-story building structures does provide significant information for structural design.

Elastic multi-degree-of-freedom (MDOF) systems are readily converted to equivalent SDOF ones by means of orthogonal mode shapes [11]. However, if inelastic response occurs, a more general approach is needed to make the conversion. For the current study, which focuses on the roof level displacement response of a building, a modified SDOF model is developed as described in this appendix.

To understand how an equivalent SDOF system represents the response of multi-story structures under earthquake excitations, the response of an idealized SDOF system is first reviewed. The equation of motion for an idealized SDOF system (Fig. B.1) under the action of earthquake ground acceleration can be written as

\[ m \ddot{u}(t) + c \dot{u}(t) + r(t) = -m \ddot{u}_g(t) \]  \hspace{1cm} (B.1)

where \( u(t) \) is the relative displacement of the mass with respect to the ground, \( r(t) \) is the resistance function of the system, \( m \) and \( c \) are mass and damping coefficients of the system, and \( \ddot{u}_g(t) \) is the ground acceleration. Denoting \( u_y \) and \( r_y \) as yielding displacement and corresponding resistance, and defining

\[
\mu(t) = \frac{u(t)}{u_y} \hspace{1cm} (a)
\]

\[
\rho(t) = \frac{r(t)}{r_y} \hspace{1cm} (b)
\]
Eq. (A.1) can be normalized to a dimensionless form as
\[
\ddot{\mu}(t) + 2\xi \omega \dot{\mu}(t) + \omega^2 \mu(t) = -\frac{\omega^2}{\eta} \ddot{u}_g(t)
\]
(B.2)

Details of the above derivation can be found in References [4,24].

Multi-story frames are commonly idealized as MDOF systems having lumped translational mass at each story level (Fig. B.2). The equation of motion for such a system can be written in matrix notation in the same format as Eq. (B.1)
\[
M\{\ddot{U}(t)\} + C\{\dot{U}(t)\} + \{R(t)\} = -M\{1\} \ddot{u}_g(t)
\]
(B.3)

where M is a diagonal mass matrix having terms \(m_i\) equal to the lumped mass at the \(i^{th}\) story level; C is a damping matrix; \(\{R(t)\}\) is the resistance vector with terms corresponding to each lateral degree of freedom; and \(\{U(t)\}\) is the displacement vector.

To convert the MDOF system cited above to an equivalent SDOF system, a fixed single deflection shape, \(\{\phi\}\), is assumed. A shape vector, which corresponds to the deflected shape under the action of statically applied lateral loads distributed in an inverse triangular pattern (Fig. B.2), is adopted in the current model. Specifically, the deflection shape corresponding to approximately 1% average drift index \((\Delta_{\text{roof}}/H)\) is used to represent the anticipated inelastic structural response to design level earthquakes. The shape vector is normalized with respect to the roof level of the frame for convenience. Therefore, at any time, \(t\), the displacements of the frame at each level can be expressed as
\[
\{U(t)\} = \{\phi\} D(t)
\]
(B.4)

where \(D(t)\) represents the magnitude of displacement at the roof level.

If the lateral loads applied to a frame are expressed as the product of a load factor \(R\) and a load distribution function \(\{P\}\), an overall load–displacement \((R–D)\) curve can be obtained from a static inelastic frame analysis (Fig. B.3). In order to idealize the \(R–D\)
curve in bi-linear form, two points corresponding to the yielding and ultimate state need to be defined.

The ultimate state is defined as the maximum expected displacement and corresponding resistance. It is recommended that the yielding point is defined as a point on the extension line of initial stiffness so that the elastic period of the system can be modeled. In addition, the elastic stiffness of the system should reflect the real working condition at the time that the earthquake occurs. Very commonly, reinforced concrete members are cracked due to the application of gravity loads and other possible loads. Therefore, the stiffness that accounts the cracking properties of each members should be used. The yielding resistance is then defined such that the area under the idealized bi-linear curve and the calculated $R-D$ curve (up to the point of maximum displacement) have equal areas, which means the energy absorption capacity is preserved. A simple bi-linear load-resistance function is defined using these definitions of yielding and ultimate states (Fig. B.3).

Substituting Eq. (B.4) into Eq. (B.3) results in

$$M\{\phi\} \ddot{D}(t) + C\{\phi\} \dot{D}(t) + \{R(t)\} = -M\{1\} \ddot{u}_p(t) \quad (B.5)$$

Pre-multiplying the transpose of the shape vector $\{\phi\}$ on both sides of Eq. (B.5), the equation becomes a scalar form as

$$M^* \ddot{D}(t) + C^* \dot{D}(t) + R^*(t) = -L^* \ddot{u}_p(t) \quad (B.6)$$

in which

$$M^* = \{\phi\}^T M\{\phi\} \quad (d)$$
$$C^* = \{\phi\}^T C\{\phi\} \quad (e)$$
$$R^*(t) = \{\phi\}^T \{R\}(t) \quad (f)$$
$$L^* = \{\phi\}^T M\{1\} \quad (g)$$

The quantities of $M^*$ can then be defined as equivalent mass, $C^*$ as equivalent damping coefficient and $R^*$ as equivalent resistance function. Dividing Eq. (B.6) by $M^*$ and noting
that

\[ \frac{C^*}{M^*} = 2\zeta \omega^* \]  \hspace{1cm} (h)

\[ \omega^{*2} = \frac{K^*}{M^*} = \frac{R_y^*}{M^*D_y} \]  \hspace{1cm} (i)

\[ \rho^*(t) = \frac{R^*(t)}{R_y^*} \]  \hspace{1cm} (j)

Eq. (B.6) can be rewritten in dimensionless form as

\[ \ddot{\tilde{\mu}}^*(t) + 2\zeta \omega^* \dot{\tilde{\mu}}^*(t) + \omega^{*2} \tilde{\mu}^*(t) = -\frac{L^*}{M^*} \frac{\omega^{*2}}{\eta^*} \frac{\ddot{u}_g(t)}{\ddot{u}_{g,\text{max}}} \]  \hspace{1cm} (B.7)

In Eq. (B.7), notations similar to those used for the SDOF systems are adopted.

Comparing Eq. (B.2) and Eq. (B.7), the only difference is the factor \(L^*/M^*\) on the right hand side. If the solution procedure for Eq. (B.2) is available, the same procedure can be applied to the solution of the equivalent system, except the ground acceleration should be scaled by that factor.

It can be noticed that the definition of the resistance function of the equivalent SDOF model presented herein does not have the physical meaning of either base shear or base overturning moment, as compared with other available models [4,49,50]. The current definition of the resistance function, \(R^*(t)\), can be explained as the dot product of the deflection shape vector and the applied load vector. This definition is preferred to either the base shear or base overturning moment, because it takes both the distribution of loads and the shape of deflection into account. In real dynamic response, the ground motion induced inertia forces and the resistance of the structure depend on the deflection shape of the structure, but this dependence is not reflected by the definition of either base shear or base overturning moment.

Because the primary purpose is to identify the displacement response, the inability of modeling either base shear or base moment will not affect the application of this model in the course of the current study. Actually, the resistance function can be written as

\[ R^*(t) = \{\phi\}^T \{R(t)\} = \{\phi\}^T \{P\} R(t) = C \times R(t) \]  \hspace{1cm} (k)
Once the value of $R^*(t)$ is determined, the base shear and base over-turning moment are readily calculated since the distribution of the lateral loads is known.

To show the validity of the equivalent SDOF model, some typical plots of roof displacement response history from inelastic frame analysis (with bi-linear moment-rotation relation for each frame member) and the displacement of equivalent SDOF system are plotted in Fig. B.4–B.6. The frame types are identified more fully in Ch. 5, and other assumptions used in Ch. 5 regarding frame analysis are also adopted.

The calculated displacement responses of the equivalent SDOF systems match, with reasonable accuracy, both the magnitude and phase of the displacement response at the roof level of the multi-story frames analyzed in this study. There are several advantages of the described equivalent SDOF model. First, the derivation of the equivalent SDOF system can be expressed mathematically in a consistent manner. Second, the period of the equivalent system approximates the elastic fundamental period of the multi-story structure in exactly the same way as the modified Rayleigh's method [4,11]. Third, the equivalent SDOF system monitors the displacement of the MDOF system at the roof level, which is more convenient than any other point. Finally, the available solution procedure for idealized SDOF systems [24] can be used without any modification. In cases that the elastic displacement response spectrum is obtained from the analysis of idealized SDOF systems, this spectrum can be modified easily (multiplied by the factor $L^*/M^*$ associated with a certain type of structural system) so that the maximum elastic displacement response at the roof level of multi-story buildings is read from the modified spectrum.
Fig. B.1 An Idealized SDOF System Subjected to Earthquake
Fig. B.2 Load Distribution and Deflection Shape for An Idealized MDOF System
Fig. B.3  A Typical Load-Displacement Relation and Bi-Linear Idealization
Fig. B.4 Displacement Response to 1978 NS Miyagi Record, Five-Story Frames
Fig. B.4  Displacement Response to 1978 NS Miyagi Record, Five-Story Frames (cont'd)
Fig. B.4  Displacement Response to 1978 NS Miyagi Record, Five-Story Frames (cont'd)
Fig. B.5 Displacement Response to 1952 S48E Santa Barbara Record, Five-Story Frames
Fig. B.5  Displacement Response to 1952 S48E Santa Barbara Record, Five-Story Frames (cont'd)
Fig. B.5 Displacement Response to 1952 S48E Santa Barbara Record, Five-Story Frames (cont'd)
Fig. B.6 Displacement Response to 1978 NS Miyagi Record, Ten-Story Frames
Fig. B.6 Displacement Response to 1978 NS Miyagi Record, Ten-Story Frames (cont'd)
Fig. B.6 Displacement Response to 1978 NS Miyagi Record, Ten-Story Frames (cont'd)